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On water waves produced by ground motions

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A linear and irrotational model is constructed to represent the formation of water waves by ground motions of a sloping bed. A survey of the constant-depth case, given first, helps in understanding the mechanism of formation, and, in this oversimplified case, wave propagation away from a source, which is usually very asymmetric. The importance of asymmetry, which may produce trapped waves, is illustrated by an estimate of the propagation in a three-dimensional case. The formation of waves by a ground motion on a slope is then studied in detail. The problem is reduced to linear integral equations of the first kind. Using an inversion technique, one constructs a source–response pair in which the source is 'δ-like' and the response is close to that which would be found if the depth was constant around the source. A general approximate solution is then derived, in both the two-dimensional and three-dimensional cases. Results for the sloping-bottom case are given for small times. They give initial values of surface displacement. They also enable one to determine the important physical parameters in the ground motion and to evaluate the efficiency of wave production.

CONTENTS

1. Introduction

The problem of water waves produced by ground motions is of theoretical as well as practical interest in several cases. The mechanisms of production can be roughly divided into three groups.

(i) The best known case is that of a source that is a seism, i.e. a very-large-scale short-duration disturbance of the ground. Tsunamis, which are thus generated, have been the object of numerous studies, both of experimental and of theoretical character (see e.g. Van Dorn 1965; Murty 1977; Hammack 1973; Hammack & Segur 1978). Throughout the mathematical studies, irrotational motion is assumed, and the source is most often represented as a disturbance of the free surface, which is readily related to the ground disturbance if an average constant depth h is taken in the source region, transients are neglected, and the linear 'small-amplitude' model is used. The propagation from the source is treated either through linear models or through 'weakly' nonlinear models in which the depth is essentially assumed to be much smaller than the 'average' wavelength, so that the limiting velocity $c = (gh)^{\frac{1}{2}}$ holds locally for the most important waves.

(ii) The other well known case is that of a centred localized source due for instance to an explosion. Again the theory starts from assuming irrotational motion. When the source is also represented as a disturbance of the free surface, the model reduces to a general model for water waves produced by explosions, which is used similarly to represent an explosion above water, inside water, or underground (Kranzer & Keller 1959; Kajiura 1963; Le Méhauté 1971; Noda 1971) and reproduces observed results with less than 30% error. This proves that the drastic approximation consisting of neglecting direct impulses to water by a submarine explosion is reliable (for rough wave-height evaluations). It is certainly much more reliable for underground motions, where the displacements and velocities are several orders smaller. Representing the ground motion by a free-surface disturbance is valid, as in case (i), if transients are neglected, the linear 'small-amplitude' model is used with constant depth, and if the ground motion is a vertical disturbance. If it is a lateral one, the equivalence is much less obvious. The propagation from the source is treated by a linear model, and the wave packet is well described by using asymptotic approximations to take into account dispersion. Nonlinear models are of course necessary near the coast.

(iii) The third case has received little theoretical study. It is that of water waves at coastal sites, due to offshore faulting, submarine slumping or underground explosions. Such phenomena have been observed in several historical events (Miloh & Striem 1976, 1978) and recently illustrated by the hydraulic phenomena observed in French Polynesia (July 1979) and in Nice (October 1979). As in the two preceding cases, the phenomenon begins with a 'long' wave, and its period, which obviously is related to the extent of the source, can be any figure between 30 s (i.e. twice that of Pacific Ocean swell) and some twenty minutes (i.e. the order of a seismic tsunami). But unlike the preceding cases, the phenomenon is very strongly asymmetrical, both in its generation (e.g. blocks falling down along the slope from the coast), and in the observation conditions (along the coast). Modelling in the 'source region' with a constant depth or using a long-wave approximation is not often appropriate, because in many cases of interest, the observed length a of the wave packet is large, but not much more than the average depth $\bar{h}$ under it, and definitely smaller than the maximum depth H at the bottom of a 10°–50° slope. A good theoretical study of two-dimensional long-wave generation, with $a \gg \bar{h}$, and neglecting dispersion effects, is given by Tuck & Hwang (1972). In addition, a few reduced-scale experiments in basins are relevant (Wiegel 1955; Prins 1958). However the relevance in case (iii) of theoretical models devised for cases (i) and (ii), is an open question. Computer calculations with asymptotic methods are more easily applied to propagation problems than to generation problems. In fact, we assume that the propagation is analysed by these means and that generation is the only new theoretical problem.

In the present paper, we try to describe the phenomena of case (iii) by means of models in which the main points are preserved, but which are sufficiently simplified that approximate formulas can be obtained to give a crude description of the wave generation. Our starting point is an irrotational linear 'small-amplitude' model solving the Cauchy problem, like the Kranzer–Keller model in case (ii). This model is applied to ground motions of a submarine hill, which is, for convenience, two-dimensional only, i.e. invariant by translation along a direction (Oz) parallel to the (straight) coast, Ox and Oy being respectively the horizontal and vertical directions normal to Oz. The three-dimensional ground motion is chosen as any small amplitude of the form $b(z)\,A(x,t)$, and the disturbance it yields is also linearly described. The model is introduced and its equations are derived in §2. Solutions at constant depth are then obtained and discussed, in order to recall some points about propagation (which we shall not study on variable depth) and to show for certain sources asymmetric effects, which give at short distances a 'diffraction lobe', and to rule out possible extensions of our study, in particular nonlinear equations. The approximate solutions for variable depth $h(x)$, with $|h'(x)| < 1$, are studied in §3. They enable us to determine the important physical parameters in the ground motion with regard to wave generation. These solutions are derived by following the intuitive idea that the generation mechanism is locally very close to the one occurring in water of constant depth equal to the local depth. The method is justified by using an unusual approach that starts from an ill-posed formulation of the problem, and regularizes it (for ill-posed problems in geophysical sciences see Sabatier 1978, 1979). For the sake of clarity, complicated details are treated in the appendix. The last section (§5) is a short discussion of the general relevance of the model with a brief survey of assumptions. Experimental results will be published later.

2. The linear model

2.1. *The model*

The straight horizontal coast is directed along the axis Oz. Let Oy be the vertical axis, pointing upward, and Ox the horizontal axis, pointing outward from the coast, and contained in the sea surface at rest. Let Oz be chosen so that $Oxyz$ is right-handed. We assume that the ground shape is described at rest by the function $y = -h(x)$, and during the ground motion by $y = -h(x)+A(x,z,t)$. We assume in addition that $h(x)$ and $A(x,z,t)$ are continuously differentiable, except for a fixed vertical boundary $(x = 0,\ y \in [-h(0), 0])$ at the coast. Now the small-amplitude and irrotational-flow linear model is described by the following equations for the velocity potential (Stoker 1957; Bouasse 1924):

$$\Delta\Phi = 0 \quad (x > 0;\ -h(x) < y < 0), \tag{2.1}$$

$$\frac{\partial\Phi}{\partial x} = 0 \quad (x = 0;\ -h(0) < y < 0), \tag{2.2}$$

$$\frac{\partial\Phi}{\partial y}+h'(x)\frac{\partial\Phi}{\partial x} = \frac{\partial A(x,z,t)}{\partial t} \quad (x > 0;\ y+h(x) \to 0^+), \tag{2.3}$$

$$\frac{\partial^2\Phi}{\partial t^2}+g\frac{\partial\Phi}{\partial y} = 0 \quad (x > 0;\ y = 0), \tag{2.4}$$

$$\frac{\partial\Phi}{\partial t} = \frac{\partial^2\Phi}{\partial t^2} = 0 \quad (x > 0;\ y = 0;\ t = 0), \tag{2.5}$$

whereas the wave amplitude is given by

$$\eta = -g^{-1}\frac{\partial\Phi}{\partial t} \quad (x > 0;\ y = 0). \tag{2.6}$$

It is convenient to continue the problem into $x < 0$ by symmetry with respect to the plane $x = 0$; $h(x)$ and $A(x)$ becoming even continuous.

Assumption 0. For the sake of simplicity, we shall also assume that $h(x)$ so defined has continuous derivatives up to and including the fourth order, that it increases steadily from $x = 0$ to $x = x_\infty$, where it reaches the value h_∞, and that it is constant, $h(x) = h_\infty$, for $x \geqslant x_\infty$. This assumption is convenient to simplify the analysis, but is not necessary. Let us introduce $\Psi(\,.\,,t) = \int_0^t \Phi(\,.\,,\tau)\,d\tau$, where Ψ is a solution of (2.1), (2.2), completed by

$$\frac{\partial\Psi}{\partial y}+h'(x)\frac{\partial\Psi}{\partial x} = A(x,z,t) \quad (y = -h(x)), \tag{2.3'}$$

$$\Psi(\,.\,,t)+g\int_0^t (t-\tau)\frac{\partial}{\partial y}\Psi(\,.\,,\tau)\,d\tau = 0 \quad (y = 0). \tag{2.4'}$$

while the surface deformation simply reduces to $[\partial\Psi(\,.\,,t)/\partial y]_{y=0}$.

There are several ways of handling this problem in order to derive an approximate solution. For a first approach, we start from two working assumptions.

Assumption A The physical problem from A to η is well-posed in $C(\mathbb{R})$ in Hadamard's sense. This means physically that a small deviation in a fall or a seism does not result into a drastic modification of the pattern of water waves, and that the whole hydraulic phenomenon is uniquely determined by giving $A(x,t)$.

Assumption B The solutions of the linear problem are close to the corresponding solutions of the physical problem.

With these assumptions, it is *sufficient* to determine a value of η that corresponds to a source $B(x,t)$ *close* to $A(x,t)$, and to estimate the accuracy of our result by evaluating the difference between A and B. A simple approach makes use of the Fourier transform. We seek a function $\Psi(x,y,z,t)$ of the form

$$\Psi(x,y,z,t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \exp\left[-2i\pi(rz+sx)\right]\tilde{\psi}(r,s,y,t)\,dr\,ds, \tag{2.7}$$

where $\tilde{\Psi}(r, s, y, t)$ is locally C_2 as a function of y, and belongs to $L_1(\mathbb{R}\times\mathbb{R})$, together with $\partial^2\tilde{\Psi}/\partial y^2$ and with $(r^2+s^2)\tilde{\Psi}$, as a function of r and s, for any physical y. Equations (2.2) and (2.1) are fulfilled if $\tilde{\Psi}(r,s,y,t)$ is an even function of s and satisfies the differential equation

$$\left[\frac{\partial^2}{\partial y^2}-4\pi^2(r^2+s^2)\right]\tilde{\Psi}(r,s,y,t) = 0. \tag{2.8}$$

Hence, taking into account (2.6), we write down $\tilde{\Psi}$ in the form $\alpha(r,s,t)\cosh[ky]+\tilde{\eta}(r,s,t)\,k^{-1}\sinh[ky]$, where

$$k = 2\pi(r^2+s^2)^{\frac{1}{2}}, \tag{2.9}$$

and $\tilde{\eta}(r,s,t)$ is the inverse Fourier transform of the surface deformation $\eta(x,y,t)$. The condition (2.4′) is fulfilled if

$$\alpha(r,s,t) = -g\int_0^t (t-\tau)\,\tilde{\eta}(r,s,\tau)\,d\tau = \tilde{\Psi}(r,s,0,t). \tag{2.10}$$

Finally, substituting Ψ into (2.3′) and Fourier-transforming the result, we obtain the integral equation

$$\begin{aligned}\int_{-\infty}^{+\infty} ds\exp\left[-2i\pi sx\right]&\Big\{(\cosh[kh(x)]+2i\pi sh'(x)\,k^{-1}\sinh[kh(x)])\,\tilde{\eta}(r,s,t)\\ &+(k\sinh[kh(x)]+2i\pi sh'(x)\cosh[kh(x)])\,g\int_0^t(t-\tau)\,\tilde{\eta}(r,s,\tau)\,d\tau\Big\}\\ &= \int_{-\infty}^{+\infty}\exp\left[2i\pi rz\right]A(x,z,t)\,dz. \end{aligned}\tag{2.11}$$

Hence, if for some source $A(x,y,t)$, (2.11) has a solution $\tilde{\eta}(r,s,t)$ such that $\tilde{\Psi}(r,s,y,t)$ fulfills the above conditions as a function of r, s and y, its Fourier transform $\Psi(x,y,z,t)$ solves the system (2.1), (2.2), (2.3′), (2.4′), and the surface deformation is the inverse Fourier transform of $\tilde{\eta}$.

For the sake of simplicity, we shall only study here the 'separable' case, where $A(x,z,t) = \mathbf{b}(z)\,A(x,t)$. Setting then $\tilde{\eta}(r,s,t) = \tilde{\mathbf{b}}(r)\,\tilde{\eta}(k,s,t)$, (see 2.9), we see that (2.11) becomes in this case

$$\begin{aligned}\int_{-\infty}^{+\infty} ds\exp\left[-2i\pi sx\right]&\Big\{(\cosh[kh(x)]+2i\pi sh'(x)\,k^{-1}\sinh[kh(x)])\,\tilde{\boldsymbol{\eta}}(k,s,t)\\ &+(k\sinh[kh(x)]+2i\pi sh'(x)\cosh[kh(x)])\,g\int_0^t(t-\tau)\,\tilde{\boldsymbol{\eta}}(k,s,\tau)\,d\tau\Big\} = A(x,t). \end{aligned}\tag{2.12}$$

In particular, if the problem is translation-invariant along Oz ('two-dimensional' case), $\tilde{\mathbf{b}}(r)$ is $\delta(r)$, and (2.12) simply reduces to

$$\begin{aligned}\int_{-\infty}^{+\infty} ds\exp\left[-2i\pi sx\right]&\Big\{(\cosh[2\pi sh(x)]+ih'(x)\sinh[2\pi sh(x)])\,\tilde{\eta}(s,t)\\ &+2\pi sg\,(\sinh[2\pi sh(x)]+ih'(x)\cosh[2\pi sh(x)])\int_0^t(t-\tau)\,\tilde{\eta}(s,\tau)\,d\tau\Big\} = A(x,t). \end{aligned}\tag{2.13}$$

2.2. *Mathematical remarks*

The conditions imposed on $\tilde{\psi}(r,s,y,t)$ restrict the set $\mathscr{A}$ of sources for which the method gives an exact solution. This is easily seen. Since $s^2\tilde{\psi}(\,.\,,s)$ must belong to $L^2(\mathbb{R})$ for any physical y, so must $s^2 \exp[kh_\infty]\,\tilde{\eta}(k(s),s,t)$. Suppose now that $h(x)$ can be continued in a domain $\mathscr{D}$ of the complex plane $\mathbb{C}$ as a holomorphic function. For any point $x \in \mathbb{C}$ such that $|\mathrm{Re}\,h(x)| + \mathrm{Im}\,x < h_\infty$, the integral on the left-hand side of (2.12) converges absolutely and uniformly, and yields an analytic function of x. There is at least one open set $\mathscr{D}$ in $\mathbb{C}$, containing one open interval of $\mathbb{R}$, and made of points satisfying this inequality. Thus, in this case, the sources $A(x,t)$ for which our method gives an exact solution cannot be chosen arbitrarily in $C(\mathbb{R})$ but only in a special set of holomorphic functions. In other words, our formulation (2.12) of the problem is ill-posed in any general set of sources like L_1 or C. This, however, need not prevent us from using this method to obtain an approximate solution for an arbitrary source A, since the assumption A and B enable us to proceed, provided that we find in $\mathscr{A}$ a good approximation of A – and we shall see that this is possible. One may wonder whether the ill-posedness of (2.12) (or (2.13)) when this integral equation is considered as a proper formulation of the linear problem (2.1)–(2.4′), is related to the fact that we have constructed harmonic functions not only in the physical range but also outside it (unless $h(x)$ is constant) or whether it is related to some intrinsic ill-posedness of the linear problem. In fact, the same equations can be obtained by using potential theory (for references see Courant & Hilbert 1962). Let us show the point on the two-dimensional case, at $t = 0^+$, for a source of the form $A(x)\,\Theta(t)$, where Θ is the Heaviside function (this case gives the result in the conditions called 'sudden approximation'). The mathematical problem is the mixed boundary-value problem defined by (2.1), (2.2), (2.3′) and (2.4′) for $t = 0$. Ψ is obtained as the potential of a single-layer distribution on $y = -h(x)$ and its symmetrical one on $y = h(x)$

$$\Psi(x,y) = \frac{1}{2}\int_{-\infty}^{+\infty} d\xi\,\sigma(\xi)\,\log\frac{(x-\xi)^2+(y-h(\xi))^2}{(x-\xi)^2+(y+h(\xi))^2}. \tag{2.14}$$

The jump-discontinuity theorem yields the condition on σ and $\eta(x,0^+)$:

$$\pi\sigma(x) + \int_{-\infty}^{+\infty} d\xi\,\sigma(\xi)\left\{\frac{h(\xi)-h(x)+h'(x)\,(x-\xi)}{[h(\xi)-h(x)]^2+(x-\xi)^2} + \frac{h(\xi)+h(x)-h'(x)\,(x-\xi)}{[h(\xi)+h(x)]^2+(x-\xi)^2}\right\} = -A(x), \tag{2.15}$$

$$\eta(x,0^+) = \left(\frac{\partial\Psi}{\partial y}\right)_{y=0} = -2\int_{-\infty}^{+\infty} d\xi\,\sigma(\xi)\,h(\xi)\,[(x-\xi)^2+h^2(\xi)]^{-1}. \tag{2.16}$$

It is possible to check that for $\sigma \in L_1(\mathbb{R})$, Ψ is such that the Dirichlet integral $\int \mathrm{grad}\,\Psi\,.\,\mathrm{grad}\,\Psi$ converges. If we seek Ψ in the class of functions for which the Dirichlet integral over the domain $-h(x) \leqslant y \leqslant 0$ converges, this solution is unique (hint: apply Green's formula). Hence, the solution of (2.1), (2.2), (2.3′), (2.4′) at $t = 0^+$ is the solution of (2.15) if it exists. Existence of a solution, and its stability with respect to perturbations of $A(x)$, are proved for the equations corresponding to (2.15) in the case of a finite basin, in §A.7 of the appendix. We shall not study the problem more generally. Notice now that from (2.16) we can calculate the inverse Fourier transform of η:

$$\tilde{\eta}(s) = -2\pi\int_{-\infty}^{+\infty} \sigma(\xi)\,\exp[-2\pi|s|h(\xi)]\,\exp[2i\pi s\xi]\,d\xi. \tag{2.17}$$

On the other hand, the result of our 'Fourier method' is given by the first part of (2.13):

$$\int_{-\infty}^{+\infty} ds \exp\left[-2i\pi sx\right]\{\cosh\left[2\pi sh(x)\right]+ih'(x)\sinh\left[2\pi sh(x)\right]\}\tilde{\eta}(s) = A(x). \quad (2.18)$$

From our analysis, we have that $\tilde{\eta}(s)$, as given by (2.17), has no reason to belong to the class of functions for which (2.18) makes sense. But suppose now that (i) $\sup|h'(x)| < 1$ and (ii) that we replace $\tilde{\eta}$ by $\exp\left[-\pi b^2 s^2\right]\tilde{\eta}(s)$, substitute it into (2.18) and take the limit as b goes to zero. For any positive b, we can commute the integrations on s and ξ and calculate exactly the integrals on s. Thanks to the inequality on $|h'|$ (see § A.2 of the appendix), these integrals go over, in the limit $b = 0$, to the sum of 'Dirac functions' and continuous functions, and integrating on ξ *we obtain the result (2.15) exactly*. Hence we learn that (2.18) can be obtained without using the Fourier method, without continuing ψ outside of the physical domain, and that it applies thus to a very large class of boundary conditions, provided this convenient regularization is applied ($\tilde{\eta}(s) \rightarrow \lim_{b\rightarrow 0} \ldots e^{-\pi s^2 b^2}\tilde{\eta}(s)$).

In the general case, using single- and double-layer potentials enables one to construct integral equations (Fredholm-type of the 2nd kind for space variables and Volterra-type in time) which yield the general equation (2.13) by the trick described above. However, they are difficult to study (because of non-compactness for unbounded domains, singularities at corners for truncated domains). This is why we prefer to use (2.13), since we only need approximate results, and with assumptions A and B, this is sufficient for deriving approximate sources and the corresponding responses.

2.3. *Fundamental sources and their approximations*

The problem is now reduced to solving (2.12) and (2.13), and evaluating the corresponding surface deformation. Since the formula (2.12) is a linear homogeneous relation between A and $\tilde{\eta}$, or η, it is sufficient to know a Green function, i.e. the response $d_0\,\eta'(A, \mathbf{b}, x_0; x, z, t)$, or its transform $d_0\,\tilde{\eta}'(A, \mathbf{b}, x_0; k, s, t)$, for ground motions of the form

$$S(A, \mathbf{b}, x_0; x, z, t) = \mathbf{b}(x_0, z)\, A(x_0, t)\, [s(x, x_0)+s(-x, x_0)], \quad (2.19)$$

where $s(x, x_0) = d_0\,\delta(x_0 - x)$, and d_0 is a certain length. The total response to a finite superposition of sources like (2.19) is $\Sigma_i\, d_i\, \eta'(A, \mathbf{b}, x_i; x, z, t)$, and that to a continuous displacement is

$$\eta(A, \mathbf{b}, x; z, t) = \int_0^{\infty} \eta'(A, \mathbf{b}, x_0; x, z, t)\, dx_0 \quad (2.20)$$

Now, our general strategy, which is inspired by inversion theory, is to seek response–source pairs in which the source $s'(x, x_0)$ is not necessarily a δ-function $\delta(x - x_0)$, but 'looks like' a δ-function, this meaning that, for any 'physical' ground motion $A(x,t)$, the function $\overline{A}(x,t) = \int s'(x, x_0)\, A(x_0, t)\, dx_0$ is close to $A(x,t)$: for instance $\|A - \overline{A}\|_{L_2}$ is small. Clearly $\|A\|^{-1} \sup \|\overline{A} - A\|$ is a way to appraise the 'δ-ness' of $s'(x, x_0)$ ('δ-ness' has been used by Backus & Gilbert 1967). But, if $s'(x, x_0)$ is the sum of a narrowly peaked function (peak at $x = x_0$, area 1, width b) and a small (maybe oscillating) function $\epsilon(x, x_0)$, $\|\epsilon\|$ and b altogether also give a measure of this 'δ-ness'. Let $\eta(A, x, x_0)$ (or its Fourier transform) be the 'response' to $s'(x, x_0)\, A(x_0, t)$. Because of the linearity, the response to $\overline{A}(x, t)$ is simply

$$\eta(x, t) = \int_{-\infty}^{\infty} \eta(A, x, x_0)\, dx_0. \quad (2.21)$$

Thanks to the assumptions A and B, the physical response is close to $\eta(x,t)$ if $\bar{A}(x,t)$ is close to $A(x,t)$ and thus the δ-ness of $s'(x,x_0)$ is a measure of quality for the approximation given by $\eta(x,t)$.

For solving (2.12) and (2.13), this approach can be used either in a general way or by considering first the part that does not contain $\int_0^t$ ('truncated equation') and then iterating on time. This method, used below, is easy to manage because of two results, which are proved for the truncated equation in §§A.1, A.2 of the appendix, and where

$$\mu(h) = \sup_{x\in\mathbb{R}} \{|h'(x)|, h_\infty |h''(x)|\}.$$

Result I. It is possible to find a pair of functions source $s_b(x,x_0)$, and response $\tilde{\eta}_b^0(s,x_0)$, depending continuously on b, in which $\tilde{\eta}_b^0 \in S(\mathbb{R})$ (S = space of C_∞ functions going to zero faster than any power of x as $x\to\infty$ – the so-called 'tempered' functions), and $\delta_b(x,x_0) = \delta(x,x_0)+\epsilon_b^0(x,x_0)$, such that $\delta_b(x,x_0)$ goes over to $\delta(x-x_0)$ as b goes to zero, and $\epsilon_b^0(x,x_0)$ goes over uniformly to a bounded function $\epsilon(x,x_0)$, whose norm as an L_2 kernel is $O(\mu(h))$. Thus, for small b, the δ-ness of this source is $O(\mu(h))$.

Result II. The limit response $\eta_0^0(x,x_0)$ is exactly that which would correspond to a δ-source at $x = x_0$ if the depth was constant and equal to $h(x_0)$. Thus, this first source–response pair corresponds to our physical intuition. From this first pair, it is possible to construct 'better' pairs with a better δ-ness for the source.

A way to do it, which can result in a complete solution of the problem, is to iterate on ϵ and on t. This is explained in §A.1 and achieved in §§A.4 and A.5 of the appendix. Here we only show our point for the iteration algorithm on ϵ. The algorithm is

$$\tilde{\eta}_b^n(s,x_0) = \tilde{\eta}_b^0(s,x_0) - \int_{-\infty}^{+\infty} \tilde{\eta}_b^{n-1}(s,x')\,\epsilon_b(x',x_0)\,dx', \tag{2.22}$$

starting at $n = 1$, stopped at $n = N$, and corresponding on one hand to the algorithm

$$\eta_b^n(x,x_0) = \eta_b^0(x,x_0) - \int_{-\infty}^{+\infty} \eta_b^{n-1}(x,x')\,\epsilon_b(x',x_0)\,dx' \tag{2.23}$$

for the response, and to the algorithm

$$\epsilon_b^n(x,x_0) = \epsilon_b(x,x_0) - \int_{-\infty}^{+\infty} \delta_b(x,x')\,\epsilon_b(x',x_0)\,dx' - \int_{-\infty}^{+\infty} \epsilon_b^{n-1}(x,x')\,\epsilon_b(x',x_0)\,dx' \tag{2.24}$$

for the source, beginning at $\epsilon_b^0(x,x') = \epsilon_b(x,x')$. For fixed N, b can be chosen so small that the free term in the right-hand side of this algorithm is negligible. Using the bound for the L_2 norm of $\epsilon(x',x_0)$, we conclude that the L_2 norm of $\epsilon_b^n(x,x_0)$ is $O(\mu^n)$. This proves our point for small $\mu(h)$. For any non-vanishing b, and any finite N, $\tilde{\eta}_b^N(s,x_0)$ is a tempered function of s, so that Φ is too. All the derivations which led from the differential problem (2.1)–(2.6) to its integral formulation are henceforth justified and $\eta_b^N(x,x_0)$ is a solution of the linear problem.

It is clear that any inversion method applied to the integral equations (2.13) or (2.18) would also yield approximate sources. It can be used as well to solve first the truncated equation and then iterate on time. In a particular problem with large $\mu(h)$ (unsmooth bottom), a well-chosen method may give better results than the present one. But it has to be specially adapted to the problem, must be dealt with by computers, and does not yield a general closed formula. But closed formulas are essential in real problems because they are the first guide for inferring source parameters from the observed wave pattern. More precise numerical methods are useful in the next step.

In the case described by (2.18), and with the assumption 0, the sequence of sources ϵ_b^n yield arbitrarily good approximations of Ψ, this reflecting the well-posedness of the problem and its regularized formulation. This well-posedness is preserved through the successive time iterations. But even if the property fails with weaker assumptions, the algorithm (2.22)–(2.24), as well as those given in the appendix, and several other inversion methods, are able to improve source–response pairs in a finite way.

3. Main features of the propagation at constant depth

In the case $h(x) \equiv h$, the integral transform in the left-hand side of (2.13) simply reduces to a Fourier transform, which can be inverted, yielding a Volterra equation for $\tilde{\eta}(k,s,t)$:

$$\cosh[kh]\left[\tilde{\eta}(k,s,t)+gk\tanh[kh]\int_0^t(t-\tau)\,\tilde{\eta}(k,s,\tau)\,d\tau\right]=\tilde{A}(s,t), \tag{3.1}$$

where $\tilde{A}(s,t)$ is the inverse Fourier transform of $A(x,t)$. Notice that physics implies $A(x,0)=[\partial A(x,t)/\partial t]_{t=0}=0$ (system at rest at $t=0$ and finite accelerations) – and similarly for $\tilde{A}(s,t)$. The Volterra equation (3.1) can be exactly solved if the source is (2.19), and we obtain

$$\tilde{\eta}'(A,x_0;\,k,s,t)=2\operatorname{sech}[kh]\cos[2\pi s x_0]\int_0^t\cos[\omega_0(t-\tau)]\frac{\partial}{\partial\tau}A(x_0,\tau)\,d\tau, \tag{3.2}$$

where

$$\omega_0(k)=(gk\tanh[kh])^{\frac{1}{2}}. \tag{3.3}$$

So as to get a feeling of the phenomenon, we may assume that the ground motion duration is so short that $\partial A(x_0,\tau)/\partial\tau$ can be replaced in (3.2) by $A(x_0)\,\delta(\tau)$. In the following, this will be called the sudden approximation. Then it follows from (3.2) that

$$\eta'(x,z,t)=4A(x_0)\int_0^\infty\int_0^\infty dr\,ds\,\tilde{\mathbf{b}}(r)\cos[2\pi rz]\{\cos[2\pi s(x+x_0)]$$
$$+\cos[2\pi s(x-x_0)]\}\operatorname{sech}[kh]\cos[(gt^2k\tanh[kh])^{\frac{1}{2}}], \tag{3.4}$$

where x_0 and A have been omitted in the labels of η'. The two terms (with $x\pm x_0$) of $\eta'(x,z,t)$ are obviously the reflected wave $\eta'_{\mathrm{R}}(x,z,t)$ and the direct wave $\eta'_{\mathrm{D}}(x,z,t)$. A simple two-parameter representation of $\mathbf{b}(z)$ is $\mathbf{b}(z)=\beta^2(\beta^2+z^2)^{-1}$, which gives $\tilde{\mathbf{b}}(r)=\beta\exp[-2\pi\beta r]$. The length β is a measure of the extent of the ground motion parallel to the coast. The total volume of the displacement is $\pi d_0 A(x_0)\,\beta$, (obviously the same for the ground motion and for the surface displacement, owing to the incompressibility assumption). Making $\beta\to\infty$, the problem goes over into a two-dimensional one, i.e. it becomes translation-invariant along Oz, and

$$\eta'_{\mathrm{D}}(x,t)=\tfrac{1}{2}A(x_0)\,g_t(x-x_0), \tag{3.5}$$

where

$$g_t(u)=2\pi^{-1}\int_0^\infty\cos[ku]\operatorname{sech}[kh]\cos[gt^2k\tanh[kh])^{\frac{1}{2}}]\,dk. \tag{3.6}$$

Let us now sketch some features of the wave propagation, so as to see where the source asymmetry may be important.

3.1. *Two-dimensional case, δ-function source and sudden approximation*

We evaluate (3.6). Let us first try using the stationary-phase approximation. The formula (3.6) contains 4 oscillating exponentials with phases $\phi^{\pm}(s,x) = -2\pi s|x_0-x| \pm \omega_0 t$. Only $\phi^{+}(s,x)$ can be stationary, and its derivative is $2\pi[t\,c(s)-|x_0-x|]$, where

$$c(s) = (2\pi)^{-1}\omega_0(2\pi s)\{(2s)^{-1}+2\pi h(\sinh[4\pi sh])^{-1}\} \tag{3.7}$$

$$\sim (gh)^{\frac{1}{2}}[1-2\pi^2 s^2 h^2] \quad (2\pi sh \ll 1) \tag{3.8a}$$

$$\sim \frac{1}{2}\left(\frac{g}{2\pi s}\right)^{\frac{1}{2}} \quad (2\pi sh \gg 1). \tag{3.8b}$$

As s goes from 0 to ∞, $c(s)$ decreases monotonically from $c_0 = (gh)^{\frac{1}{2}}$ to 0. Thus the stationary-phase equation $|x-x_0| = tc(s)$ has no solution for $|x-x_0| > c_0 t$, and only one, s_0, for smaller $|x-x_0|$. In the stationary-phase method, one concludes that the propagated waves reach x at $t = c_0^{-1}|x-x_0|$. For larger times, $c(s)$ is replaced by $c(s_0)+(s-s_0)c'(s_0)$, and the Gaussian integral is calculated between $-\infty$ and $+\infty$. Results are reliable if both $\phi^{+}(s_0,x)$ and $ts_0^2|c'(s_0)|$ are large. These conditions, together with the approximations (3.8*a*) or (3.8*b*), hold in some parts of the wavetrain, for instance in the following.

3.1.1. *Wavetrain tail* ($gt^2|x-x_0|^{-1} \gg 1$; $c_0 t|x-x_0|^{-1} \gg 1$). The method yields wave amplitudes smaller than $0{\cdot}4\,A(x_0)\,(gt^2)^{\frac{1}{2}}|x-x_0|^{-\frac{3}{2}}$.

3.1.2. *First waves at large distances.* Let $\xi = 1-|x-x_0|(c_0 t)^{-1}$. Using (3.8*a*) and the stationary-phase method is possible if the two conditions $\xi \ll 1$ and $\xi^{\frac{3}{2}}c_0 t/h \gg 1$ are simultaneously satisfied. Then $\pi s_0 h \sim (\frac{1}{2}\xi)^{\frac{1}{2}}$, and

$$\eta'_{\mathrm{D}}(x,t) \sim \tfrac{1}{2}\pi^{-\frac{1}{2}}(A(x_0)\,[(\tfrac{1}{2}\xi)^{\frac{1}{2}}h\,c_0 t]^{-\frac{1}{2}}\cos\left[\frac{2\sqrt{2}}{3}\frac{c_0 t}{h}\,\xi^{\frac{3}{2}}-\tfrac{1}{4}\pi\right]. \tag{3.9}$$

The striking difference between this result and the corresponding one of the Kranzer–Keller model is its dependence on the distance d from the source, i.e. $|x-x_0|$ (or $c_0 t$). This dependence is here as $d^{-\frac{1}{2}}$, whereas in the K.K. axial model it is as d^{-1}, an obvious consequence of the problem symmetries.

3.1.3. *First waves at short distances and short times.* The method does not apply to deriving them, although they are the most-important ones. A more general asymptotic method using Airy functions can work, as will be seen in §3.2.1. We give here only a very rough estimate, which is obtained by neglecting dispersion ($c(s) = c_0$). The result is

$$\eta'_{\mathrm{D}}(x,t) \sim \tfrac{1}{4}A(x_0)\,h^{-1}\,\mathrm{sech}\,[\tfrac{1}{2}\pi h^{-1}(|x-x_0|-c_0 t)]\,\Theta(|x-x_0|-c_0 t), \tag{3.10}$$

where Θ is the Heaviside function. This leading term carries more than half of the source volume. It shows that even a δ-function source yields but a very smooth water deformation.

Combining (3.8), (3.9), and (3.10), we can 'follow' the phenomenon and see the single wave transformed into many, according to the general dispersion that is implied by $c(s)$. Near the coast we also have to take the reflected waves into account.

3.1.4. *The reflected waves.* The reflected term η'_{R} is given by (3.5), with $x+x_0$ instead of $x-x_0$. Far from the coast, the direct and the reflected wavetrains are thus completely shifted. Near the coast (because of our total reflection assumption), the wavetrains are superposed, so that the reduced amplitude at $x = 0$ and $t = c_0^{-1}x_0$ is twice the direct one.

3.2. *Two-dimensional case, arbitrary source, and sudden approximation*

For $A(x,t) = A(x)\Theta(t)$, the response is obtained by integrating (3.4). This yields in the three-dimensional and two-dimensional cases

$$\eta(x,z,t) = 4\int_0^{\infty}\int_0^{\infty} dr\,ds\,\tilde{b}(r)\tilde{A}(s)\cos[2\pi rz]\cos[2\pi sx]\operatorname{sech}[kh]\cos[t\omega_0(k)], \quad (3.11)$$

$$\eta(x,t) = 2\int_0^{\infty} ds\,\tilde{A}(s)\cos[2\pi sx]\operatorname{sech}[2\pi sh]\cos[t\omega_0(2\pi s)] \quad (3.12a)$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty} d\xi\,A(\xi)\,g_t(x-\xi). \quad (3.12b)$$

Let us choose
$$A(x) = L\left(F\left[\frac{x-x_1}{l}\right]+F\left[\frac{x+x_1}{l}\right]\right),$$

where F is smooth, even and Gaussian-looking. Then η again can be written as the sum of a direct term η_D (function of $x-x_1$) and a reflected one η_R (function of $x+x_1$), with

$$\eta_D(x,t) = 2lL\int_0^{\infty} ds\operatorname{sech}[2\pi sh]\cos[2\pi s|x-x_1|]\tilde{F}(ls)\cos[t\,\omega(2\pi s)], \quad (3.13)$$

where $\tilde{F}$ is the Fourier transform of F. The difference between (3.13) and (3.5) lies in the 'non-oscillating' function, $\operatorname{sech}[2\pi sh]$, being now multiplied by $\tilde{F}(ls)$. Hence the analysis is not different. The cases that correspond to those of §§3.1.*1* and 3.1.2 are ruled by the same formulas, except that $A(x_0)$ is replaced by $lL\tilde{F}(ls_0)$. Thus the wave amplitude for first waves at large distances depends on ξ as $\xi^{-\frac{1}{3}}\tilde{F}[\frac{1}{2}\xi)^{\frac{1}{3}}l/\pi h]$. This function is not usually monotone, so that the main wave at large distances is not usually the first one, and its rank increases with the distance $|x-x_1|$. Let us now study more deeply the following case, which corresponds to that of §3.1.3:

3.2.1. *First waves at short distances and short times.* New points appear if the source width l is definitely larger than h. Because of F-smoothness, we expect that $\tilde{F}$ vanishes rapidly for values of $s \gtrsim l^{-1}$, so that the validity range of the non-dispersive approximation increases considerably. It yields

$$\eta_D(x,t) \sim \tfrac{1}{2}L[G(|x-x_1|-c_0t)+G(|x-x_1|+c_0t)], \quad (3.14)$$

where G is the Fourier transform of $\tilde{G}(s) = l\operatorname{sech}[2\pi sh]\tilde{F}(l)$. $G(x)$ reduces to $F(x/l)$ in the limit $h/l \to 0$. So as to check the validity of (3.14) at intermediate times, we can use in (3.13) the formula (3.7) and the special form, which is often an acceptable approximation, $\tilde{G}(s) \sim G\exp[-2\pi s^2H^2]$. We obtain

$$\eta_D(x,t) = LG[f(ct+X,t)+f(ct-X,t)], \quad (3.15)$$

where $X = |x-x_1|$, and

$$f(\Delta,t) = \frac{1}{2}\int_{-\infty}^{+\infty}\exp[\tfrac{4}{3}i\pi^3s^3h^2c_0t-2\pi^2s^2H^2-2i\pi s\Delta]\,ds$$

$$= (4h^2c_0t)^{-\frac{1}{3}}\exp\left[\frac{1}{3}\frac{H^3}{h^2c_0t}\left(\frac{H^3}{h^2c_0t}-3\frac{\Delta}{H}\right)\right]Ai\left[\frac{H}{(4h^2c_0t)^{\frac{1}{3}}}\left(\frac{H^3}{h^2c_0t}-\frac{2\Delta}{H}\right)\right]. \quad (3.16)$$

If simultaneously $H > 1{\cdot}5\,(h^2c_0t)^{\frac{1}{3}}$ and $H > 2\Delta$, the asymptotic form of the Airy function that corresponds to the 'rainbow dark side' can be used, giving

$$f(\Delta,t) \sim \tfrac{1}{2}(2\pi)^{-\frac{1}{2}}H^{-1}\exp\left[-\frac{\Delta^2}{2H^2}\right](1+\tfrac{1}{2}\Delta H^{-4}h^2c_0t); \quad (3.17)$$

had the approximation (3.14) been used, the last factor would have been missing – an error of only a few per cent. For larger Δ, only the first oscillations on the 'rainbow bright side' of the Airy function are different from their estimates obtained through the stationary-phase method. In conclusion, the larger width of a (smooth) source may be sufficient to justify a shallow-water approximation, even at intermediate times, provided that the beginning of the wavetrain only is considered (i.e. up to a distance of the order of the source width). For $h \ll l$, the result (3.14), where $G(x) \sim F(x/l)$, is readily obtained from (3.12*b*), since $g_t(u)$ is then a peaked function of width $\sim h$, and can therefore be replaced by $\delta(u-ct)$. This remark will be used in the three-dimensional case.

3.3. *Two-dimensional case without sudden approximation*

From (3.2), we easily obtain

$$\eta(x,z,t) = 4\int_0^\infty \int_0^\infty dr\,ds\,\mathfrak{b}(r) \cos[2\pi rz] \cos[2\pi sx] \operatorname{sech}[kh] \int_0^t d\tau \cos[(t-\tau)\,\omega_0(k)] \frac{\partial}{\partial\tau} \tilde{A}(s,\tau), \quad (3.18)$$

$$\eta(x,t) = 2\int_0^\infty ds \cos[2\pi sx] \operatorname{sech}[2\pi sh] \int_0^t \cos(t-\tau)\,\omega(s)] \frac{\partial}{\partial\tau} \tilde{A}(s,\tau)\,d\tau, \quad (3.19)$$

where $\omega(s) = \omega_0(2\pi s)$. Suppose now that the ground motion lasts between $t = 0$ and $t = t_0$, and that we are interested by the phenomenon after $t = t_0$. Integrating by parts, we get $\eta = \eta_{\mathrm{P}} + \eta_{\mathrm{T}}$, with

$$\eta_{\mathrm{P}}(x,t) = 2\int_0^\infty ds \cos[2\pi sx] \cos[\omega(s)\,(t-t_0)]\,\tilde{G}(s,t_0)\,ds, \quad (3.20)$$

$$\eta_{\mathrm{T}}(x,t) = -2\int_0^\infty ds \cos[2\pi sx]\,\omega(s) \int_0^t \sin[\omega(s)\,(t-\tau)]\,\tilde{G}(s,\tau)\,d\tau, \quad (3.21)$$

where $\tilde{G}(s,\tau) = \operatorname{sech}[2\pi sh]\,\tilde{A}(s,\tau)$. η_{P} is due to the permanent ground displacement and is of the form studied in §3.2, with $t-t_0$ instead of t. η_{T} is due to transients. For a smooth function $A(x,t)$, $\tilde{A}(s,t) \operatorname{sech}[2\pi sh]$ is negligible beyond $s \sim (2\pi h)^{-1}$. Hence, if the ground-motion duration t_0 is so small that $gt_0^2 \ll h$, the values of $\omega(s)\,\tau$ that contribute to the integral are small, and we can replace $\sin[\omega(s)\,(t-\tau)]$ by $\sin[\omega(s)\,(t-t_0)]$, obtaining the rough approximation

$$\eta_{\mathrm{T}}(x,t) \sim 2\frac{\partial}{\partial t}\int_0^\infty ds \cos[2\pi sx] \cos[\omega(s)\,(t-t_0)] \int_0^{t_0} \tilde{G}(s,\tau)\,d\tau. \quad (3.22)$$

The condition $gt_0^2 < h$ means physically that during the ground motion, gravity waves do not have time to propagate to $O(h)$. The right-hand side of (3.22) can be evaluated like the permanent displacement. In particular, if $\tilde{G}(s,t) = \tilde{G}(s)\,T(t)$, and if the permanent displacement does not vanish, $\eta_{\mathrm{T}} \sim -t_1\,\partial\eta_{\mathrm{P}}/\partial t$, where $t_1 = [T(t_0)]^{-1}\int_0^{t_0} T(\tau)\,d\tau$. It is not surprising that η_{T} is related to acceleration. If t_1 is not too large, the effect of the transients is simply to shift the apparent beginning of the phenomenon, since $\eta(x,t) \sim \eta_{\mathrm{P}}(x,t-t_1)$. Compared with a 'sudden ground motion', the total time shift is $t_0 + t_1$.

3.4. *Three-dimensional case*

According to (3.11), the 'direct term' of the response to $A(x_0)\,[\delta(x-x_0)+\delta(x+x_0)]$ is

$$\eta'_{\mathrm{D}}(x,z,t) = 4\,A(x_0)\int_0^\infty\int_0^\infty dr\,ds\,\mathfrak{b}(r)\cos\,[2\pi rz]$$
$$\cos\,[2\pi sX]\,\mathrm{sech}\,[kh]\cos\,[t\omega_0(k)], \quad (3.23)$$

where $X = |x-x_0|$. Using (3.6), we easily derive

$$\eta'_{\mathrm{D}}(x,z,t) = -A(x_0)\int_0^\infty dr\,\mathfrak{b}(r)\cos\,[2\pi rz]\int_X^\infty du\left[\frac{d}{du}\,g_t(u)\right]J_0\,[2\pi r(u^2-X^2)^{\frac{1}{2}}]. \quad (3.24)$$

Since $g(u)$ vanishes for $u > c_0 t$, so does η'_{D} for $X > c_0 t$. By integrating by part, we can write $\eta'_{\mathrm{D}}(x,z,t)$ as a superposition of two-dimensional amplitudes, which is convenient for analysis. The same remark applies to the response to an extended source:

$$\eta(x,z,t) = -\int_0^\infty dr\,\mathfrak{b}(r)\cos\,[2\pi rz]\int_{-\infty}^{+\infty} d\xi\,A(\xi)$$
$$\int_0^\infty dv\,J_0[2\pi rv]\,\frac{d}{dv}\,g_t([v^2+(x-\xi)^2]^{\frac{1}{2}}). \quad (3.25)$$

We shall illustrate this formula only by an example using the special forms (already introduced) $\mathfrak{b}(r) = \pi\beta\exp\,[-2\pi\beta r]$ and $A(x) = L[F(X_+/l)+F[X_-/l)]$, with $X_\pm = |x\pm x_1|$. Thus

$$\eta_{\mathrm{D}}(x,z,t) = -\tfrac{1}{2}\beta L\int_{-\infty}^{+\infty} F\left(\frac{w}{l}\right)dw\int_0^\infty dv\;\mathrm{Re}\,[v^2+(\beta+iz)^2]^{-\frac{1}{2}}$$
$$\frac{d}{dv}\,g_t([v^2+(X^- - w)^2]^{\frac{1}{2}}) \quad (3.26a)$$

$$= \tfrac{1}{2}\beta L\int_{-\infty}^{+\infty} F\left(\frac{w}{l}\right)dw\left\{g_t(X-w)\;\mathrm{Re}\,[\beta+iz]^{-1}\right.$$
$$\left.+\int_0^\infty dv\,g_t([v^2+(X_- - w)^2]^{\frac{1}{2}})\,\frac{d}{dv}\,\mathrm{Re}\,[v^2+(\beta+iz)^2]^{-\frac{1}{2}}\right\}. \quad (3.26b)$$

The reflected term is similar, with X_+ instead of X_-. Simple results can be obtained if $|X| \gg l$, and $h \ll l \ll \beta$. In this non-dispersive range, $g_t(u)$ is a peaked function centred at $u = ct$, width $\sim h$, and $F(w/l)$ is a peaked function centred at $w = 0$, width $\sim l$. Two parts of the pattern are easy to see (we assume $X_- < ct$).

(i) $|z| \lesssim \frac{1}{2}\beta$. The second term in (3.26*b*), compared with the first one, is of order $l\beta ct\,(c^2t^2+\beta^2-X_-^2)^{-\frac{3}{2}}$, and is therefore negligible. The first term is dominant, and hence the motion is almost two-dimensional in this domain, which defines the axial part of the 'emission lobe'.

(ii) $|z| \gtrsim 2\beta$. The right-hand side of (3.26*b*) has noticeable values at two places. The first one is $ct = X$, where the first term has its maximum value $\frac{1}{2}Ll\beta^2(\beta^2+z^2)^{-1}$. The second one is $ct = [X_-^2-\beta^2+z^2]^{\frac{1}{2}}$, where the peaks of F, g_t, and d Re [ ]$/dv$, coincide in the second term of (3.26*b*), which is then equal to $\frac{1}{8}Llct\beta^{-\frac{1}{2}}z^{-\frac{3}{2}}$. Clearly, the relative importance of these two wave amplitudes changes as t increases. It is even possible to find domains in the non-dispersive range in which the second term is dominant and has a value of the opposite sign. Thus the results are complicated. However, the general decrease of the crest envelope for fixed X_- is relatively slow, very roughly like $z^{-\frac{1}{2}}$ as long as X_- lies between l and β^2/l. Along the coast, the presence of the reflected term doubles the effects, so that, for x_1 between l and β^2/l, one may have a significant amplitude up to z equal to a few β. Notice that the width of the wave travelling in the direction Ox (first term of 3.26*b*) is of order l. That of the radial wave arriving at $ct = [x^2 - \beta^2 + z^2]^{\frac{1}{2}}$ is of order β in typical cases. If the depth varies rapidly after $x \sim x_2$, say, this long wave can be strongly reflected by the slope, so that the energy that is not emitted along the direction Ox remains confined between the reflecting coast and the reflecting slope (typical values $h = 100$ m, $l = 400$ m, $\beta = 2000$ m, $x_1 = 800$ m, $x_2 = 1500$ m). Besides, even in the constant-depth case, nonlinear effects in wave propagation can alter the dispersion effects for these long waves, so that the slow decrease may be continued for larger ct. Needless to say, at large distances, asymmetry effects are reduced (and rapidly overwhelmed by refraction effects on variable depth).

4. The sloping bottom case

We give in §§A.2–A.4 of the appendix the sequences of successive approximations for solving (2.12) and (2.13), with iterations both on $\epsilon(x, x_0)$ and on t. If all assumptions are valid, these sequences, as well as the regularizing trick, can give arbitrarily good approximations of Ψ. However, we only use the first or the second iterated term, as given in (A 36). According to our analysis, they correspond to sources (2.19) of non-vanishing δ-ness, so that the approximate sources they can build are usually smoother than the exact ones. Thus, for certain sources one cannot expect more than a rough approximation of the results.

In the zeroth-order approximation, we keep only the two first terms in the right-hand side of (A 36). It physically means that each localized perturbation of the bottom does not 'know' that the bottom is not horizontal. The next terms take care of the reflection on the slope (relative $O(\mu(h))$). A point must be made. Up to the relative $O(t^2\mu(h))$, taking into account this reflection is equivalent to making a time-independent source correction. For an estimate at small times, especially for phenomenological sources depending on few parameters, it is better to keep only the zeroth-order term and to remodel the source to account for the slope correction. This correction has been done implicitly below.

4.1. *Sudden approximation: general formulas*

At time $t = 0^+$, for $A(x_0, 0^+) \neq 0$, we obtain from (A 36) first terms (or from (3.5)) for an extended source

$$\eta_D(x,t) \sim 2\int_0^\infty d\sigma\,(x_0)\int_0^\infty ds \cos\,[2\pi s(x-x_0)]\,\mathrm{sech}\,[2\pi s h(x_0)]\cos\,[\omega(x_0, 2\pi s)\,t], \quad (4.1)$$

where $\omega(x_0, k) = \{gk \tanh\,[kh(x_0)]\}^{\frac{1}{2}}$, and $d\sigma(x_0) = A(x_0)\,dx_0$ is the differential cross-section of the ground perturbation. $\eta_R(x,t)$ is given by the same formula, with $\cos\,[2\pi s(x+x_0)]$ instead of $\cos\,[2\pi s(x-x_0)]$.

Our derivations and estimates hold only for small t (more precisely $\frac{1}{2}gt^2 \lesssim h_0$), both because higher orders are neglected and because the flanks of the δ-like sources are negligible if the propagation time of information coming from them is much larger than t. On the other hand, the results should be precise within these conditions. They can then be used as an initial surface displacement for propagation studies.

The above derivations and approximations are extended in §A 6 of the appendix to the three-dimensional (separable) case, yielding from (A 45) (or (3.4)) the approximate value

$$\eta_D(x,z,t) \sim 4\int_0^\infty A(x_0)\,dx_0\int_0^\infty\int_0^\infty dr\,ds\,\mathfrak{b}(r)\cos\,[2\pi rz]\cos\,[2\pi s(x-x_0)]$$
$$\times\,\mathrm{sech}\,[kh(x_0)]\cos\,[\omega(x_0,k)\,t], \quad (4.3)$$

which is valid in the sudden approximation. With $\tilde{\mathfrak{b}}(r) = \pi\beta \exp[-2\pi\beta r]$, and $dV(x_0) = \pi\beta A(x_0)\,dx_0$, (4.3) reads

$$\eta_D(x,z,t) = 4\int_0^\infty dV(x_0)\int_0^\infty\int_0^\infty dr\,ds\,\exp[-2\pi\beta x]\cos[2\pi rz]\cos[2\pi s(x-x_0)] \times \operatorname{sech}[kh(x_0)]\cos[\omega(x_0,k)]. \quad (4.4)$$

4.2. *Two-dimensional analysis: sudden approximation*

So as to determine the important parameters, we have to study $\eta(x,t)$ (or $\eta(x,z,t)$) in the first moments, the problem at later times being more a problem of propagation on water of variable depth. Let us study them first in the sudden approximation. From (A 19a) and (A 36), we see that the first deformation of the water surface is

$$\eta(x,0) = \frac{1}{2}\int_0^\infty [h(x_0)]^{-1}\left\{\operatorname{sech}\frac{\pi(x-x_0)}{2h_0} + \operatorname{sech}\frac{\pi(x+x_0)}{2h_0}\right\} d\sigma(x_0). \quad (4.5)$$

One can check on this formula that the total cross-section σ of the surface wave $\left(\int_0^\infty \eta(x,0)\,dx\right)$ is equal to the total cross-section $\left(\int_0^\infty d\sigma(x_0)\right)$ of the ground displacement (incompressibility). We make several remarks.

(i) A displacement $d\sigma(x_0)$ contributes the wave amplitude at a fixed point x with a weight that cannot be larger than $[h(x_0)]^{-1}$, and decreases exponentially for large $(x-x_0)$. Hence it is not correct to predict the effects of a ground motion by giving only the displaced volume $\left(\int_0^\infty d\sigma(x_0)\right)$. If the ground motion is due to blocks falling down or a loop sliding along the slope, we model it by a crater near the coast (small $h(x_0)$) and a bump far away from the coast (large $h(x_0)$). Then we see that the first hydraulic effects on the coast are due to the cratering. If $d\sigma(x_0)$ is a very smooth function of x_0, extended over the whole slope, the initial surface displacement near the coast will typically be a negative half-oscillation of width $\sim h_M$, (maximum height $\sim 2\sigma/h_M$, strongly assymetric towards the coast), approximately centred at the 'crater centre', i.e. halfway on the slope (depth h_M). The initial surface displacement away from the coast will be a positive half-oscillation of width $\gtrsim 2h_M$, initial height $\sim \sigma/2h_M$. This positive half-oscillation will arrive at the coast later, and, because of its 'wavelength', it should be strongly reflected in any model. This is why we assumed total reflection in our linear model. However, in true phenomena, the propagation would also show several nonlinear features.

So as to get a more precise feeling of the $t = 0$ situation in a particular case, one can consider a ground displacement in which $\Delta\sigma$ is extracted from the ground around x_0^{i}, with a lateral extent that is small compared to h_0, and put at x_0^{f}. The corresponding amplitude is

$$\eta_{\Delta\sigma}(x,0) \approx \tfrac{1}{2}\Delta\sigma[-r(x,x_0^{\mathrm{i}}) + r(x,x_0^{\mathrm{f}})], \quad (4.6)$$

where

$$r(x,x_0) = [h(x_0)]^{-1}\left\{\operatorname{sech}\frac{\pi(x-x_0)}{2h_0} + \operatorname{sech}\frac{\pi(x+x_0)}{2h_0}\right\}. \quad (4.7)$$

A general displacement of matter is then conveniently done by superposing various displacements Δr of this kind. η is then given by the sum

$$\eta(x,0) \approx \tfrac{1}{2}\sum_i \Delta\sigma_i[-r(x,x_0^{\mathrm{i}}) + r(x,x_0^{\mathrm{f}})]. \quad (4.8)$$

Going to the limit of a smooth superposition of these displacements, one can justify the qualitative description that has been given above.

(ii) In the sudden approximation, η is stationary at $t = 0$. The surface deformation energy is the (two-dimensional) potential energy $W = \frac{1}{2}\rho_0 g \int_0^\infty \eta^2(x,0)\,dx$. The maximum energy E available because of the ground displacement is readily related to the area $\Delta\sigma$ that has been displaced and to its depth shift. We evaluate the rate of efficiency W/E in several cases.†

Case 1: Narrow displacement. We assume that a prism of area $\Delta\sigma = Ad$ (width $d \ll h_i = h(x_i)$) is taken away at x_i and goes instantaneously to x_f ($\Delta x = x_f - x_i$). First notice that our model holds only if the energy release $E = \rho g\,\Delta\sigma\,(h_f - h_i)$ is larger than the energy $E_{\min} = \rho g\, d^{-1}(\Delta\sigma)^2$ that is necessary to create a hole in the ground at x_i and the corresponding bump at x_f. Now W can be calculated by using (4.6), with the result

$$W \approx \tfrac{1}{8}\sqrt{2}\,\rho_0 g(\Delta\sigma)^2 \left\{\frac{1}{h_i} + \frac{1}{h_f} - \frac{4(h_i+h_f)}{(h_i+h_f)^2 + \frac{1}{8}\pi^2(x_i-x_f)^2} + \frac{h_i}{h_i^2 + \frac{1}{8}\pi^2 x_i^2} + \frac{h_f}{h_f^2 + \frac{1}{8}\pi^2 x_f^2} - \frac{4(h_i+h_f)}{(h_i+h_f)^2 + \frac{1}{8}\pi^2(x_i+x_f)^2}\right\}. \tag{4.9}$$

If x_i is far enough from the coast, we can neglect the last three terms, which correspond to reflected waves, and are both small and not very relevant (in the sense that our approximations may have introduced larger errors). Thus, with $\Delta h = h_f - h_i$,

$$W \approx 0{\cdot}17\,\rho_0 g(\Delta\sigma)^2 \frac{(h_i+h_f)\,[(\Delta h)^2 + 1{\cdot}24(\Delta x)^2]}{h_i h_f\,[(h_i+h_f)^2 + 1{\cdot}24(\Delta x)^2]}. \tag{4.10}$$

The rate $r = W/E$ is always small. If $\Delta h \ll \Delta x \lesssim h_M = \frac{1}{2}(h_i + h_f)$, r is smaller than $0{\cdot}09\,\rho_0/\rho$ for $E = E_{\min}$, viz 5% for $\rho_0 \sim 1$, $\rho \sim 1{\cdot}8$. However, this would correspond to small slopes, and small energy releases. In most physical cases ($h'_M \in (0{\cdot}1, 0{\cdot}8)$), and even for $\Delta x \lesssim h_M$, E is much larger than $E_{\min}$. Then the rate is asymptotic to $0{\cdot}18\,\Delta\sigma\, h_M^{-3}\,\Delta x\, h'^{-1}_M\ \ (1 + 0{\cdot}8h'^{2}_M)\,\rho_0/\rho$. For $\Delta x = h_M = 400$ m, $\Delta\sigma = 10000$ m^2, $h'_M = 0{\cdot}5$, one gets $r = 1{\cdot}5$%! Besides, in many cases, Δx is very large, much larger than h_f sometimes. Then it follows from (4.10) that the rate of efficiency is asymptotic to

$$r \sim 0{\cdot}17\,\frac{\Delta\sigma}{h_i h_f}\,\frac{h_f + h_i}{h_f - h_i}\,\frac{\rho_0}{\rho} \quad (\Delta x \gg h_i,\ E \gg E_{\min}) \tag{4.11}$$

whereas $W \sim 0{\cdot}17\,\rho_0 g(\Delta\sigma)^2[h_i^{-1} + h_f^{-1}]$. Hence, for a narrow loop slide from h_i to h_f, with $h_f \gg h_i$, it is certainly more physical to appraise it by the number $0{\cdot}17\,(\Delta\sigma)^2/h_i h_f$ than to do it by $\Delta\sigma$, as it is usually done.

Case 2: Wide displacement. We assume that the amplitude of the displacement is very gently varying with depth, so that it is reasonably represented by a function $A(x) = d\sigma/dx$ so smooth that it does not vary much on $\Delta x \sim h(x)$. Thus the coefficient of $A(x_0)$ in (3.5) behaves like a δ-function and $\eta(x,0) \sim A(x)$. Then, if $A(x) \sim A \sin(2\pi x/L)$ for $x \leqslant L$, 0 elsewhere, $W \sim \frac{1}{4}\rho_0 g A^2 L$. The energy release of gravity forces in the ground displacement is $E \sim \frac{1}{2}\rho g A L \Delta H$, where ΔH is the depth shift, and has to be larger than A. Hence it seems that the rate of efficiency could be important and go up to 25%. However, this is misleading, because the only case in which this approximation may be valid, *together* with the sudden approximation, is that of a tsunami created by a very-large scale earthquake. But the gravity forces work in the permanent displacement is then but a very small part (a few thousandths)

† Several authors have given the efficiencies corresponding to experimental studies. Efficiency definitions involve a lot of arbitrariness.

of the energy release in the earthquake! In the case of a landslide, the ground displacement velocity is always much smaller than $(gh)^{\frac{1}{2}}$ and the sudden approximation fails utterly.

4.3. *Two-dimensional analysis and giving up the sudden approximation*

Again we use the lowest-order approximation and we neglect the reflected waves. The formula that corresponds to (4.2) is

$$\eta_{\mathrm{D}}(x,t) \approx 2\int_0^\infty dx_0 \int_0^\infty ds\,\cos\,[2\pi s(x-x_0)]\,\mathrm{sech}\,[2\pi s h(x_0)]$$
$$\int_0^t \cos\,[\omega_0(t-\tau)]\,\frac{\partial}{\partial\tau}A(x_0,\tau)\,d\tau. \quad (4.12)$$

There are two main ways of giving up the sudden approximation.

First way. We introduce the displacement as a whole ('separable' function $A(x_0,t)$), and we obtain the permanent displacement either progressively or after oscillations. This is what we did in §3.3, and the analysis we could give here is not really different. In particular, the amplitude of the transients is an increasing function of the displacement derivatives. For a very smooth displacement, the transients hardly contribute to the hydraulic phenomenon. The best known physical cases are the ground displacements due to explosions or to seisms. On real physical examples we have been able to see that even a transitory displacement four times larger than the permanent one yields a smaller contribution to the water waves.

Second way. So as to give a better fit for underwater landslides, we use the progressive function

$$A(x_0,t) = \Delta S[G(x_0-x_1-vt)-G(x_0-x_1)]\,\Theta(t), \quad (4.13)$$

where Θ is the Heaviside function, $G(x)$ is a bump-like function that has a width a and a total area equal to 1, e.g. $G(x) = a^{-2}x\exp\,[-x/a]\,\Theta(x)$.

Clearly, $A(x_0,t)$ is a fairly good representation of a piece of ground taken away between x_1 and x_1+2a and travelling along the slope, away from the coast, with velocity v. Putting an additional factor $\Theta(t_0-t)$ stops the motion at t_0.

Case 2(a) $a \ll h(x_1)$. It is reasonable to approximate G by a δ-function: say $G(x_0-x_1) \approx \delta(x_0+x_1)$. Let us then insert (4.13) into (A 36), and assume that we study the phenomenon for $gt^2 \lesssim h(x_1)$. We obtain for η'_{D} the following estimate:

$$(\Delta S)^{-1}\,\eta'_{\mathrm{D}}(x,t) = [\gamma_0(x,x_1+vt)-\gamma_0(x,x_1)] - \int_0^t (t-\tau)\,[\gamma_1(x,x_1+v\tau)$$
$$-\gamma_1(x,x_1)]\,d\tau - \int_{-\infty}^{+\infty} d\xi\,\gamma_0(x,\xi)\,[\epsilon(\xi,x_1+vt)-\epsilon(\xi,x_1)] + O\left(\frac{\mu(h_1)\,gt^2}{h_1}\right), \quad (4.14)$$

where $$\gamma_0(x,x_1) = \int_{-\infty}^{+\infty} \exp\,[-2i\pi sx]\,\tilde{\gamma}_0(s,x_1)\,ds = \tfrac{1}{2}\,[h(x_1)]^{-1}\mathrm{sech}\,\frac{\pi X}{2h(x_1)},$$

$$\gamma_1(x,x_1) = \int_{-\infty}^{+\infty} \exp\,[-2i\pi sx]\,\omega_1^2(s,x)\,\tilde{\gamma}_0(s,x_1)\,ds = \tfrac{1}{2}g[h(x_1)]^{-2}\left(1+X\frac{\partial}{\partial X}\right)\mathrm{sech}\,\frac{\pi X}{2h(x_1)}$$
$$X = x-x_1.$$

We have only kept the two first terms in the right-hand side of (4.14) for a rough evaluation (the remainders being useful to check its validity). The extreme opposite case to the sudden approximation is $v \ll c_1$ (with $c_1 = [gh(x_1)]^{\frac{1}{2}}$). It yields

$$\eta'_{\mathrm{D}}(x,t) \sim v\,\Delta S\left[t\frac{\partial}{\partial x_1}\gamma_0(x,x_1) - \frac{1}{G}t^3\frac{\partial}{\partial x_1}\gamma_1(x,x_1)\right], \quad (4.15)$$

which at $x = x_1$ is maximum for $gt^2 = h(x_1)$ and then approximately equal to $-0{\cdot}33\, h_1' v/h_1 c_1$. Values of the maximum in neighbouring points are also proportional to $h_1' v/h_1 c_1$, with somewhat-different coefficients.

Case 2(b) Very smooth G with $a \gg h(x_1)$ and very small h_1'. We shall make drastic approximations. First we assume that the very-long-period (small-s) components are dominant (this was done in §§3.1.3 and 3.2.1). Thus (4.12) can be written in the form $\eta_D(x,t) = \eta_D(x,t,c) + \eta_D(x,t,-c)$,

$$\eta_D(x,t,c) = \int_0^\infty dx_0 \int_0^{+} \frac{\partial}{\partial \tau} A(x_0,\tau)\, d\tau \int_0^\infty ds \frac{\cos\,[2\pi s(x - x_0 + ct - c\tau)]}{\cosh\, 2\pi sh}$$

$$= \frac{1}{4}\int_0^\infty dx_0 \int_0^t \frac{\partial}{\partial \tau} A(x_0,t)\, h^{-1}\, \mathrm{sech}\left[\frac{\pi}{2h}(x - x_0 + ct - c\tau)\right] d\tau. \qquad (4.16)$$

Inserting (4.13) into (4.16), and replacing the sech by a δ-function since its width h is much smaller than a, we obtain after adding $\eta_D(x,t,-c)$, and to the second order in v/c, a very simple result

$$\eta_D(x,t) \sim \frac{1}{4}\frac{\Delta S\, v}{c}\,[G(x - x_1 - ct) - G(x - x_1 + ct)] \qquad (4.17)$$

in which a trough going towards the coast and a hump going away are superposed, with the local velocity c. They are built by interference of the locally created waves and the propagated ones. Quantitatively, this formula is not justified, unless h' is very small. So as to justify the method devised in the appendix, one must not observe the phenomenon outside of a range $\sim h(x_0)/h'(x_0)$ around a source at x_0. So a and ct must be smaller than h_1/h_1', with $a \gg h_1$ and $ct > a$ when the two waves are separated. Thus (4.17) may hold at most in a narrow range around x_1. If it does, the rate W/E_{kin} is simply $\frac{1}{8}(\rho_0/\rho)\,\Delta S/ah_1$, which is qualitatively consistent with experimental results.

Taking into account higher orders in the developments gives more reliable results but at the price of much more complicated formulas.

4.4. *Three-dimensional analysis*

From (2.12) and (A 46) we obtain the formula

$$\eta_D(x,z,t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dr\, ds\, dx_0 \exp\,[-2i\pi(rz + sX)]\,\bar{\mathbf{b}}(r)\,\mathrm{sech}\,[kh(x_0)]$$
$$\left\{A(x_0,t) - gk\tanh\,[kh(x_0)]\int_0^t (t-u)\,A(x_0,u)\,du - \int_{-\infty}^{+\infty} A(\xi,t)\,\varepsilon(x_0,\xi)\,d\xi\right\}$$
$$+ O(\bar{h}'^2) + O\left(\left(\frac{gt^2}{h}\right)^2\right) + O\left(\frac{\bar{h}'\, gt^2}{h}\right). \qquad (4.18)$$

We shall study three cases.

4.4.1. *Sudden approximation: source elongated in the z-direction.* Keeping only the first term in (4.18) is equivalent to integrating (3.23) over x_0 (direct term). We make here only two remarks.

(i) the weight of a source localized at x_0 being $[h(x_0)]^{-1}$, there is an apparent diminution of the source in the direction normal to the coast. Hence the asymmetry assumed in §3.4 is more easily achieved.

(ii) the initial surface depression being elongated along the coast, we learn from the constant-depth case that one can see at small distances on one hand a small decreasing along the coast; on the other hand one sees a concentration of energy in the direction normal to the coast, viz a diffraction lobe. In the case of a slope, energy trapping along the coast will be helped both because waves are reflected from the slope and because the initial larger apparent length of the wave along the coast makes a faster propagation into this direction and helps the formation of a trapped wave.

4.4.2. *Sudden approximation: localized source.* For a source of volume ΔV, half-width α along x, β along z, it is convenient to set

$$A(x_0,t)\,\bar{\mathfrak{b}}(r) = \Theta(t)\,\Delta V\alpha^{-1}\exp\left[-\pi\left(\frac{x_0-x_1}{\alpha}\right)^2\right]\exp\left[-\pi r^2\beta^2\right], \tag{4.19}$$

where α and β are assumed to be small compared with h_0. Besides, we are interested only in the surface shape near the maximum deformation, which is usually close to $x = x_1$ and $z = 0$ at $t = 0$, so that one can assume $R^2 = X^2 + z^2 \ll h_1^2$. Now let us first suppose that α and β vanish, so that the first two terms in (4.18) reduce to

$$\begin{aligned}\eta_D(x,z,t) &\approx 4\Delta V\int_0^\infty dr\int_0^\infty ds\cos\left[2\pi sX\right]\cos\left[2\pi rz\right]\operatorname{sech}\left[kh_1\right]1-\tfrac{1}{2}gt^2k\tanh\left[kh_1\right]) \\ &= (2\pi)^{-1}h_1^{-2}\Delta V\int_0^\infty du\,J_0\left[\frac{uR}{h_1}\right]u\operatorname{sech}\left[u\right]\left[1-\tfrac{1}{2}u\tanh\left[u\right]\frac{gt^2}{h_1}\right].\end{aligned} \tag{4.20}$$

Clearly, for $R \ll h_1$, $J_0(uR/h_1)$ can be replaced by $1-\frac{1}{4}u^2R^2/h_1^2$, giving the bottom of the deformation by the two first terms of its R^2 expansion.

We learn in (4.20) that, for a very rough approximation, sech u could be replaced by any function $f(u)$ going very rapidly to zero beyond $u = 1$ and such that

$$\int_0^\infty u\operatorname{sech}u\,du = \gamma = \int_0^\infty du\,f(u)\,du\int_0^\infty u^2\operatorname{sech}u\tanh u\,du = \int_0^\infty u^3f(u)\,du.$$

Such a function is $\gamma\exp\left[-\frac{1}{2}u^2\right]$. Thus, for non-vanishing α and β, let us replace sech $[kh(x_0)]$ in (4.18) by $\gamma\exp\left[-\frac{1}{2}k^2h_1^2\right]\left[1-k^2(x_0-x_1)\,h_1\,h'_1\right]$. It turns out that the term containing ε, (evaluated by means of (A 44)) vanishes. The two first terms yield, for small α/h_1 and small β/h_1,

$$\eta_D(x,z,t) \sim \gamma\frac{\Delta V}{2\pi h_1^2}\tilde{\alpha}\tilde{\beta}\exp\left[-\frac{\tilde{R}^2}{2h_1^2}\right]\left[1-\tilde{\alpha}^2h'_1\frac{X\alpha^2}{h_1^3}\left(2+\frac{\tilde{R}^2}{2h_1^2}\right)-\frac{gt^2}{h_1}\left(1-\frac{\tilde{R}^2}{2h_1^2}\right)\right], \tag{4.21}$$

where $X = x - x_1$, $\tilde{\alpha} = (1+\alpha^2/2\pi h_1^2)^{-\frac{1}{2}}$, $\tilde{\beta} = (1+\beta^2/2\pi h_1^2)^{-\frac{1}{2}}$, and $\tilde{R}^2 = \tilde{\alpha}^2X^2+\tilde{\beta}^2z^2$. The main term appears as an overall coefficient because the corrections were obtained by differentiating it. Throughout these derivations, we neglect the variations of $\tilde{\alpha}$ and $\tilde{\beta}$, and consider α^2/h_1^2 and β^2/h_1^2 as infinitesimal parameters. One could write consistently $\tilde{\alpha} \sim 1-(4\pi)^{-1}\alpha^2/h_1^2$ for instance. We see that the value at $z = 0$, $x = x_1$ is multiplied by $\tilde{\alpha}\tilde{\beta}$, which is smaller than 1 and decreases with the source spreading. The slope correction is always small, whereas the time evolution becomes essential for $t \sim (h_1/g)^{\frac{1}{2}}$. Compared with the two-dimensional result (4.6), we see that (4.20) or (4.21) show not only the expected additional h_1^{-1} but also an extra factor γ/π, which is smaller than 1.

4.4.3. *Slow displacement: localized source.* We keep only the first two terms in (4.18), and we insert (4.13), with

$$\bar{\mathfrak{b}}(r)\,G(x_0) = \Delta V\alpha^{-1}\exp\left[-\left(\frac{(x_0-x_1)}{\alpha}\right)^2\right]\exp\left[-\pi r^2\beta^2\right]. \tag{4.22}$$

Let $F(h_1)$ be the main term in (4.21) (we shall neglect the variations of $\tilde{\alpha}$ and $\tilde{\beta}$ with h_1). The response to (4.13) is approximately

$$F(h(x_1+vt))-F(h_1)+2gh_1\int_0^t(t-\tau)\frac{\partial}{\partial(h_1^2)}\left[F(h(x_1+vt))-F(h_1)\right].$$

To the first order in h_1', this yields

$$\eta_{\mathrm{D}}(x,z,t) \sim 2h_1' \frac{vt}{h_1}\left\{-1+\frac{\tilde{R}^2}{2h_1^2}+\frac{1}{3}\frac{gt^2}{h_1}\left[\left(-1+\frac{\tilde{R}^2}{2h_1^2}\right)^2+\left(1-\frac{R^2}{h_1^2}\right)\right]\right\} F(h_1). \quad (4.23)$$

In particular, at $x = x_1$, $z = 0$, η_{D} is an extremum for $gt^2 \sim \frac{1}{2}h_1$, and the extremum is equal to

$$\eta_{\mathrm{D\,ext}} \sim -\tfrac{4}{3}h_1' \frac{v}{c\sqrt{2}}\gamma\frac{\Delta V}{2\pi h_1^2}\tilde{\alpha}\tilde{\beta}. \quad (4.24)$$

Notice that for fixed slope and limit speed v this decreases like $h_1^{-\frac{5}{2}}$.

5. Final remarks

5.1. *Conclusions*

We gave the initial surface displacement of a body of water following underwater ground motions. Depth is variable, with slope everywhere smaller than 45°. It is assumed that the slope is constant on lines parallel to the coast.

(i) The relative effectiveness of a source localized at $x = x_0$ (two-dimensional case), $x = x_0$, $z = 0$ (three-dimensional case) in its contribution to the amplitude is proportional to $[h(x_0)]^{-1}\,dS$ (two-dimensional case), to $[h(x_0)]^{-2}\,dV$ (three-dimensional case), with a smaller coefficient in the three-dimensional case.

(ii) The initial displacement lateral size is of the order of $h(x_1)$ if the source width a is much smaller than $h(x_1)$ (x_1 = source-centre abscissa), but is of the order of a if $a \gg h(x)$ throughout the source.

(iii) If a localized ground displacement starts sliding from $x = x_1$ with a constant velocity $v \ll c = [gh(x_1)]^{\frac{1}{2}}$, the generated wave amplitude is roughly proportional to $dS\,h_1'\,h_1^{-1}\,v/c_1$ (two-dimensional case), $dV h_1' h_1^{-2} v/c_1$ (three-dimensional case).

(iv) If a source is extended offshore, with its length (parallel to the coast) larger than its width, this asymmetry is increased by depth effects. It results *at small distances* in a diffraction lobe, which may generate trapped waves or energy concentrations in the propagation that follows.

(v) If the permanent displacement is reached more slowly, and with a separable time dependence (e.g. explosions, earthquakes), the hydraulic effects of transients are usually much smaller than those of permanent displacements.

(vi) The effects of ground motions that are equivalent to instantaneous transfer of ΔS from x_1 to x_2 depend essentially on $h(x_1)$, $h(x_2)$, ΔS, and the average slope. The rate of efficiency is almost always smaller than 5 %.

(vii) Slow and very wide progressive ground motions (e.g. landslides; for a recent review see Slingerland & Voight 1979) are beyond the scope of the present paper because the wave-propagation problems and wave-formation problems become combined. However, a very rough estimate shows that the rate of efficiency is still smaller than in other cases, unless the ground velocity is of the order of the wave velocity.

5.2. *Comparison with experimental results*

As has been mentioned, there is qualitative agreement with published experimental results made in the laboratory. The agreement with stereo films of real water waves due to ground motions in comparable cases is acceptable. New experimental results will be published soon. In most cases the main waves are somewhat smaller than those predicted here.

5.3. *Criticism of theory and possible improvements*

(i) The linear treatment of sources is usually good for explosions and earthquakes. In the cases of rockfalls and landslides it is certainly poor, but may be no more so than any source representation by an analytically tractable model.

(ii) Nonlinear effects are often important near the shore line and may modify a trapped wave.

(iii) The problem of slow progressive motions on variable depth cannot be managed quantitatively by our theory but other approximations (see Slingerland & Voight 1979) are appropriate.

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Appendix

A 1. *Preparation*

In both the two-dimensional and the three-dimensional cases (for $r = 0$), the study of (2.12), (2.13) reduces to that of

$$\lim_{b\to 0}\left\{\int_{-\infty}^{+\infty} e^{-\pi s^2 b^2}\, ds \left[K(x,s)\,\tilde{\eta}(s,t)+\int_0^t d\tau(t-\tau)\,L(x,s)\,\tilde{\eta}(s,\tau)\right]\right\} = A(x,t), \quad \text{(A 1)}$$

where we have taken into account the regularization schemes, and

$$2\pi s\, K(x,s) = i\frac{\partial}{\partial x}\{\exp\,[-2i\pi sx]\cosh\,[2\pi s h(x)]\}, \quad \text{(A 2a)}$$

$$2\pi s\, L(x,s) = i\frac{\partial}{\partial x}\{\omega^2(s,x)\exp\,[-2i\pi sx]\cosh\,[2\pi s h(x)]\}, \quad \text{(A 2b)}$$

$$\omega^2(s,x) = 2\pi s g\tanh\,[2\pi s h(x)]. \quad \text{(A 2c)}$$

The equation obtained by keeping only the first term in the left-hand side of (A 1) will be called the truncated equation. Its solution at $t = 0^+$ is the 'sudden approximation'. Now, for a constant depth $h(x_0)$, (A 1) has an exact solution. This suggests introducing, *for a given* x_0

$$\tilde{\gamma}(s,x_0,t) = \tilde{\eta}(s,t)+\omega^2(s,x_0)\int_0^t d\tau\,(t-\tau)\,\tilde{\eta}(s,\tau). \quad \text{(A 4)}$$

Inserting (A 3) into (A 1) yields

$$\lim\left\{\int_{-\infty}^{+\infty} ds\, e^{-\pi s^2 b^2}\left[K(x,s)\,\tilde{\gamma}(s,x_0,t) + \int_0^t d\tau\,\omega_0^{-1}\sin\,[\omega_0(t-\tau)]\,k(x,x_0,s)\,\tilde{\gamma}(s,x_0,\tau)\right]\right\} = A(x,t), \quad \text{(A 4)}$$

where ω_0 stands for $\omega(s,x_0)$, and

$$2\pi s k(x,x_0,s) = i\frac{\partial}{\partial x}\{\omega^2(s,x)-\omega^2(s,x_0)]\exp\,[-2i\pi sx]\cosh\,[2\pi s h(x)]\}. \quad \text{(A 5)}$$

The truncated equation is the same one in (A 4) and (A 1). We shall first study this equation. In §A2, we evaluate

$$I_0^{\mathrm{D}}(x, x_0) = \lim_{b\to 0}\int_{-\infty}^{+\infty} ds\, e^{-\pi s^2 b^2}\, K(x,s)\, \mathrm{sech}\,[2\pi s h(x_0)]\, \exp\,[2i\pi s x_0], \qquad \text{(A 6)}$$

the reflected term $I_0^{\mathrm{R}}(x, x_0)$ being obtained by making $x_0 \to -x_0$. The integral (A 6) for $b > 0$, say, $I_0^{\mathrm{D}}(x, x_0)$, is a tempered function of x. We shall show that for $b \to 0$, I_0^{D} and I_0^{R} contain a δ-function and a remainder. We prove in §A 3 that this remainder is bounded in such a way that the iteration scheme described by (2.22)–(2.24) works even in the limits $b \to 0$ and $N \to \infty$. Thus we are able to construct a 'resolvent kernel' for the truncated equation. At this point, we can try to 'improve' the pair-source response for the whole equation (A 1) or (A 4) by successive approximations mainly taking into account the operators on x. This was done in an earlier version of this work. However, it is easier to iterate on time. We show in §A 4 how this iteration series converges, and in §A 5 we give its first terms, which are the only ones to be used in the present paper. In §A 6, we sketch in few lines the method used in the three-dimensional problem, and in §A 7 we sketch some well-posedness proofs for the linear problem.

A 2. *δ-ness of $I_0(x, x_0)$*

Let us introduce the sequence $h_n = (2n+1)\,h(x_0)$, and, for any $h(x) \in [h_n, h_{n+1}[$, we replace cosh $[2\pi s h(x)]$ and sinh $[2\pi s h(x)]$ in (A 6) by using the following formulas, which are readily proved by induction:

$$\cosh H_1\, \mathrm{sech}\, H_0 = 2\{\cosh\,(H_1 - H_0) - \cosh\,(H_1 - 3H_0) + \ldots (-1)^n$$
$$\times \cosh\,(H_1 - (2n+1)\,H_0)\} + (-1)^{n+1}\cosh\,[H_1 - (2n+2)\,H_0]\,\mathrm{sech}\, H_0, \qquad \text{(A 7)}$$

$$\sinh H_1\, \mathrm{sech}\, H_0 = 2\{\sinh\,(H_1 - H_0) - \sinh\,(H_1 - 3H_0) + \ldots (-1)^n$$
$$\times \sinh\,(H_1 - (2n+1)\,H_0)\} + (-1)^{n+1}\sinh\,[H_1 - (2n+2)\,H_0]\,\mathrm{sech}\, H_0, \qquad \text{(A 8)}$$

where H_1, H_0 stand for $2\pi s h(x)$, $2\pi s h_0$. Now, the last-term contribution to the integral (A 6) is uniformly convergent for any $h(x)$ of the open interval $]h_n, h_{n+1}[$, and any $b \geqslant 0$. Let us assume in the following that $|h'(x)| < C < 1$ for any x. The other terms involved in (A 6) can be exactly calculated, for any $b > 0$, as a finite sum of terms of the form

$$I^{\pm}(X, \Delta_q(x)) = 2\,[1 \pm ih'(x)]\int_{-\infty}^{+\infty}\exp\,[-\pi s^2 b^2 \pm 2\pi s \Delta_q(x) - 2i\pi s X]\,ds$$
$$= 2\,[1 \pm ih'(x)]\,b^{-1}\exp\,[-\pi b^{-2}(X \pm i\Delta_q(x))^2], \qquad \text{(A 9)}$$

where $\Delta_q(x) = h(x) - (2q+1)\,h(x_0)$, and X is $x - x_0$, or $x + x_0$. The behaviour of (A 9) depends on q. Both X and $\Delta_0(x)$ have $\pm x_0$ as a common root. It is clear that $I^{\pm}(X, \Delta_0)$ does not converge in any space of functions of a real variable. Besides, for any function of x that belongs to $C_1(\mathbb{R})$ and has finite support in $[x_0, \infty)$ one can write

$$\lim_{b\to 0}\int_{x_0}^{\infty} I^{\pm}(x - x_0, \Delta_0(x))\,f(x)\,dx = 2\lim_{b\to 0} b^{-1}\int_{x_0}^{\infty}\exp\,[-\pi b^{-2}(x - x_0 \pm i\Delta_0(x)^2]$$
$$\times \{f(x_0) + (x - x_0)\,f'[x_0 + \theta(x - x_0)]\}\,(1 \pm ih'(x))\,dx, \qquad \text{(A 10)}$$

where θ is an unknown number between 0 and 1, depending on x. Because of our assumption on h', and if F' is an upper bound of $|f'|$, the part containing f' in (A 10) is absolutely bounded by

$$2F' \lim_{b\to 0} \int_{x_0}^{\infty} \exp\,[-\pi b^{-2}(1-C)^2(x-x_0)^2]\,(x-x_0)\,dx,$$

which goes to zero with b. The other part involves the integral of $\exp\,[-\pi b^{-2}z^2]$ in the complex plane on the countour $z = (x-x_0) \pm i\Delta_0(x)$, starting at $x = x_0$, and is therefore equal to

$$2f(x_0) \lim_{b\to 0} b^{-1} \int_{x_0}^{\infty} \exp\,[-\pi b^{-2}(x-x_0)^2]\,dx,$$

i.e. $f(x_0)$. Hence, for functions of $C_1(\mathbb{R})$ with finite support in $[x_0, \infty)$, the function of x $I^{\pm}(x-x_0, \Delta_0)$ goes over to $\delta_{x_0}(x)$ as $b \to 0$. Now let us assume that $h'(x)$ is itself differentiable. Permuting x and x_0, and replacing $h'(x)$ inside the integral by $h'(x_0) + (x-x_0)\,h''[x_0 + \theta(x-x_0)]$, we prove in the same way that for functions of $C_1(\mathbb{R})$ with support in $[0, x]$, the function $I^{\pm}(x-x_0, \Delta_0)$ of x_0 goes over to $\delta_x(x_0)$ as $b \to 0$. Transformation of these remarks are trivial if we use the symmetry convention described in §1 and allow negative x. We conclude that, provided convenient conditions are respected in the limit processing, $\lim_{b\to 0} I^{\pm}(X, \Delta_0) = \delta(X)$.

Assume now that $q \neq 0$. It follows from the monotone-slope assumption that $|x_q| > x_0$. Let us study the case $x_q > x_0$. It follows from the $|h'(x)| \leqslant C < 1$ assumption that $|\Delta_q(x)| < C|x-x_q|$. We are interested only in the values of $x : x \geqslant x_q$. Now, it follows from (A 9) that

$$|I_b^{\pm}(X, \Delta_q(x))| \leqslant 2\sqrt{2}\,b^{-1} \exp\,[-b^{-2}(X^2 - C^2(x-x_q)^2)],$$

and the argument in the exponential is lower than its value for $x = \frac{1}{2}(x_1 + x_q)$, viz (in the case $X = x - x_0$, which yields there the lower value) $\frac{1}{4}(x_0 - x_q)^2\,(1-C^2)$. Hence, for any $x \geqslant \frac{1}{2}(x_0 + x_q)$, in particular $x \geqslant x_q$, $I_b^{\pm}(X, \Delta_q(x))$ goes uniformly to zero as b goes to zero. The results are readily extended to the case $x \leqslant -\frac{1}{2}(x_0 + x_q)$. The same uniform convergence to zero holds also for $I_b^{\pm}(X, \Delta_0(x))$ if $|x| > \frac{1}{2}(x_0 + x_1)$ and to

$$\int_{|x| > \frac{1}{2}(x_0 + x_1)} |I_b^{\pm}(X, \Delta_0(x)|^2\,dx.$$

Let us now use these results for calculating (A 6). For $0 \leqslant h(x) < h_0$, only the last terms of (A 7) and (A 8) remains. Thanks to our simplifying assumption of a monotone $h(x)$, we can write

$$\begin{aligned} I_0(x < x_0, x_0) = A(x_0, t) \int_{-\infty}^{+\infty} &\{\exp\,[-2i\pi s(x-x_0)] \\ &+ \exp\,[-2i\pi s(x+x_0)]\}\,\mathrm{sech}\,[2\pi s h(x_0)] \\ &\times \{\cosh\,[2\pi s h(x)] + ih'(x)\,\sinh\,[2\pi s h(x)]\}\,ds \\ = A(x_0, t)\,\{&\mathrm{Re}\,[(1 + ih'(x)g[x - x_0 + ih(x)]] \\ &+ \mathrm{Re}\,[(1 + ih'(x))g[x + x_0 + ih(x)]]\} \quad (x < x_0), \end{aligned} \tag{A 11}$$

where $g(z)$ is defined by

$$g(z) = \int_{-\infty}^{+\infty} \mathrm{sech}\,[2\pi s h_0]\,\exp\,[2i\pi s z]\,ds$$

$$= (2h_0)^{-1}\,\mathrm{sech}\,[\pi(2h_0)^{-1}z] \tag{A 12a}$$

$$= (2h_0)^{-1} \left\{ -\cosh \frac{\pi(x \pm x_0)}{2h_0} \sin \frac{\pi\Delta_0(x)}{2h_0} + i\,\sinh \frac{\pi(x \pm x_0)}{2h_0} \cos \frac{\pi\Delta_0(x)}{2h_0} \right\}^{-1} \tag{A 12b}$$

for $z = x \pm x_0 + i\,h(x)$ and $\Delta_0(x) = h(x) - h(x_0)$.

For $h_0 \leqslant h(x) < 3h_0$, we use (A 7) and (A 8) with $n = 0$. The term containing $\cosh(H_1 - H_0)$ readily yields $A(x_0, t)[\delta(x+x_0)+\delta(x-x_0)]$, whereas the last term is calculated exactly as in (A 11), with the result

$$-A(x_0,t)\{\mathrm{Re}\,[(1-ih'(x))g[x-x_0+i(2h(x_0)-h(x))]]$$
$$+\mathrm{Re}\,[(1-ih'(x))g[x+x_0+i(2h(x_0)-h(x))]]\} \quad (x_0 \leqslant x < x_1). \tag{A 13}$$

If we notice that $2h(x_0)-h(x) = h(x_0)-\Delta_0(x)$, we can write down (A 13) as a function of $\Delta_0(x)$. Using (A 12*b*), and noticing that generally $\mathrm{Re}\,[(1+iC)(-a+ib)^{-1}] = \mathrm{Re}\,[-(1-iC)(a+ib^{-1})]$, we find that (A 13) is the analytic continuation of (A 11) through x_0.

More generally, for $h_n \leqslant h(x) < h_{n+1}$, or $x_n \leqslant x < x_{n+1}$, we obtain

$$i\lim_{b\to 0}(-1)^{n+1}\int_{-\infty}^{+\infty}\exp[-\pi s^2 b^2]\frac{\partial}{\partial x}\{(2\pi s)^{-1}\exp[-2i\pi s(x-x_0)]$$
$$\times\cosh[2\pi s((2n+2)h(x_0)-h(x))]\,\mathrm{sech}\,2\pi s\,h(x_0)\}\,dx$$
$$=(-1)^{n+1}\,\mathrm{Re}\,\{(1-i\,h'(x))\,g[x-x_0+i((2n+2)\,h(x_0)-h(x))]\}, \tag{A 14}$$

and, of course, a similar result with x_0 instead of $-x_0$. Writing down (A 14) as a function of $\Delta_0(x)$ yields again the closed form that has been obtained for $h(0) \leqslant h(x) < 3h(x_0)$. If the maximum depth is H, and $E(H/h(x_0)) = 2N+1$, we can stop at $n = N$. Referring to (A 6), we conclude that the source that would yield the left-hand side of (A 6) is

$$s_0(x,x_0) = \delta(x,x_0)+\epsilon_0(x,x_0), \tag{A 15a}$$

with

$$\epsilon_0(x,x_0) = \frac{-\cosh\dfrac{\pi(x-x_0)}{2h_0}\sin\dfrac{\pi\Delta_0(x)}{2h_0}+h'(x)\sinh\dfrac{\pi(x-x_0)}{2h_0}\cos\dfrac{\pi\Delta_0(x)}{2h_0}}{2h(x_0)\left\{\sin^2\dfrac{\pi\Delta_0(x)}{2h_0}+\sinh^2\dfrac{\pi(x-x_0)}{2h_0}\right\}}. \tag{A 15b}$$

Notice that the reflection makes $h(x) = h(-x)$, $h'(x) = -h'(-x)$, $s(-x,x_0) = s(x,-x_0)$. $\delta(x-x_0)$ is the limit, as b goes to zero, of $\delta_b(x,x_0) = \mathrm{Re}\,I_b^-(x-x_0,\Delta_0(x))$. $\epsilon_0(x,x_0)$ is the limit of $\epsilon_b(x,x_0)$, which is the sum of two terms: for $x\in[x_n,x_{n+1}[$, the first one is the term under 'lim' in the left-hand side of (A 14), say $\bar\epsilon_b(x,x_0)$. The second one, which appears for $n>0$, is the sum of $\mathrm{Re}\,I_b^-(x-x_0,\Delta_p(x))$ for $p = 1, 2, \ldots n$. As we have seen, this second term goes uniformly to 0 as $b\to 0$, whereas $\bar\epsilon_b(x,x_0)$ goes uniformly to $\epsilon_0(x,x_0)$.

A 3. *Improvement of the δ-ness*

It is easy to see in (A 14) and (A 15) that $\bar\epsilon_b(x,x_0)$ and $\epsilon_0(x,x_0)$ are identically zero for $|x|$ *and* $|x_0|$ larger than x_∞, since $h(x) = h(x_0) = h_\infty$. On the other hand, $\epsilon_0(x,x_0)$ (and thus $\epsilon_b(x,x_0)$ for small b) can be bounded by means of (A 15). Let

$$u = \frac{\tfrac12\pi(x-x_0)}{h(x_0)},\quad \bar h' = \sup_{x\in\mathbb{R}}|h'(x)|,\quad \bar h'' = \sup|h_\infty h''(x)|,\quad \mu(h) = \sup(\bar h',\bar h'').$$

It is easy to derive the bounds, valid for ϵ_0 or ϵ_b with b small enough:

$$2|\epsilon(x,x_0)| \leqslant \left\{\begin{matrix}\dfrac{\bar h'\,|u\cosh u+\sinh u|}{h_0\sinh^2 u}\\[2ex] \dfrac{\bar h'\,|u\cosh u-\sinh u|}{h_0\sinh^2 u}+\dfrac{\bar h'' u^2\cosh u}{h_0\sinh^2 u}\end{matrix}\right\} \tag{A 16a}$$

$$\|\epsilon\|^2 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dx\,dx'\,\epsilon^2(x,x') \leqslant C\mu^2(h), \tag{A 16b}$$

where C is some number.

Now, let $\varepsilon_b^N(x,x_0)$ be the Nth iterated kernel defined from $\varepsilon_b^0 = \epsilon_b$ by

$$\varepsilon_b^N(x,x_0) = \int_{-\infty}^{+\infty} dx_1\,\varepsilon_b^{N-1}(x,x_1)\,\epsilon_b(x_1,x_0). \tag{A 17}$$

Then Cauchy's inequality yields $\|\varepsilon_b^N\| \leqslant \|\epsilon_b\|^{N+1}$. Suppose we want to obtain an approximate solution of

$$\int_{-\infty}^{+\infty} ds\,K(x,s)\,\tilde{\eta}(s) = B(x), \tag{A 18}$$

knowing that for

$$\tilde{\eta}_0^b(s,x_0) = e^{-\pi s^2 b^2}\,\mathrm{sech}\,[2\pi s h(x_0)]\,\exp\,[2i\pi s x_0], \tag{A 19a}$$

$$\int_{-\infty}^{+\infty} ds\,K(x,s)\,\tilde{\eta}_0^b(s,x_0) = \delta_b(x,x_0)+\epsilon_b(x,x_0). \tag{A 19b}$$

We can proceed by constructing a sequence of functions

$$\begin{aligned} b_N(x) &= B(x) - \sum_0^{N-1} (-1)^n \int_{-\infty}^{+\infty} dx_0\,\epsilon_b^n(x,x_0)\,B(x_0) \\ &= B(x) - \int_{-\infty}^{+\infty} dx_0\,\epsilon_b(x,x_0)\,b_{N-1}(x_0), \end{aligned} \tag{A 20}$$

which yields a sequence of sources

$$\tilde{\eta}_N(s) = \int_{-\infty}^{+\infty} \tilde{\eta}_b^0(s,x_0)\,b_N(x_0)\,dx, \tag{A 21}$$

and a corresponding sequence of responses

$$\begin{aligned} \int_{-\infty}^{+\infty} ds\,K(x,s)\,\tilde{\eta}(s) &= \int_{-\infty}^{+\infty} dx_0\,\delta_b(x,x_0)\,B(x_0) - \int_{-\infty}^{+\infty} dx_0\,\delta_b(x,x_0) \\ &\times \int_{-\infty}^{+\infty} dx_1\,\epsilon_b(x_0,x_1)\,b_{N-1}(x_1) + \int_{-\infty}^{+\infty} dx_0\,\epsilon_b(x,x_0)\,b_N(x_0). \end{aligned} \tag{A 22}$$

For $B(x) = \delta(x-x_0)$, the response (A 22) can be written in the form $\delta_b(x,x_0)+\epsilon_b^n(x,x_0)$, where ϵ_b^n is given by (2.24). It is easy to express ϵ_b^n in terms of the ε_b^n, and to prove (2.23). For any $B(x)$, if $C\mu^2(h) < 1$, one can go to the limits $b = 0$ and $N = \infty$, for which the right-hand side of (A 22) reduces to $B(x)$. Hence (A 21), in the limits $N\rightarrow\infty$ and $b = 0$, gives the exact solution of (A 18), and $\lim_{N\rightarrow\infty} b_N(x) = b(x)$ is the solution of

$$b(x) = B(x) - \int_{-\infty}^{+\infty} dx'\,\epsilon(x,x')\,b(x'), \tag{A 23a}$$

$$\text{i.e.}\quad b(x) = B(x) - \int_{-\infty}^{+\infty} dx'\,R(x,x')\,B(x'). \tag{A 23b}$$

The (truncated) Neuman series of the resolvent R appears in the right-hand side of (A 20). Clearly we can also use this resolvent for transforming (A 1) into the integral equation

$$\tilde{\eta}(s',t) = \int_{-\infty}^{+\infty} dx'\,\tilde{\eta}_0^0(s',x')\left[B(x',t) - \int_{-\infty}^{+\infty} R(x',x_1)\,B(x_1,t)\,dx_1\right], \tag{A 24}$$

where

$$B(x,t) = A(x,t) - \lim_{b\to 0}\int_{-\infty}^{+\infty} ds\, e^{-\pi s^2 b^2}\int_0^t d\tau\,(t-\tau)\,L(x,s)\,\tilde{\eta}(s,t). \qquad (\text{A } 25)$$

Equation (A 4) can also be transformed into (A 24), with $\tilde{\gamma}(s', x_0, t)$ instead of $\tilde{\eta}(s', t)$, and $B(x,t)$ replaced by

$$B(x,x_0,t) = A(x,t) - \lim_{b\to 0}\int_{-\infty}^{+\infty} ds\, e^{-\pi s^2 b^2}\int_0^t d\tau\,\frac{\sin[\omega_0(t-\tau)]}{\omega_0}\,k(x,x_0,S)\,\tilde{\gamma}(s,x_0,\tau) \quad (\text{A } 26)$$

We can also insert (A 26) into (A 24), obtaining an integral equation for $B(x, x_0, t)$:

$$B(x,x_0,t) = A(x,t) - \lim_{b\to 0}\int_{-\infty}^{+\infty} ds\, e^{-\pi s^2 b^2}\int_0^t d\tau\,\omega_0^{-1}\sin[\omega_0(t-\tau)]\,k(x,x_0,s)$$

$$\times\int_{-\infty}^{+\infty} dx'\,\tilde{\eta}_0^0(s',x')\left[B(x',x_0,\tau) - \int_{-\infty}^{+\infty} R(x',x_1)\,B(x_1,x_0,\tau)\,dx_1\right]. \quad (\text{A } 27)$$

The 'reference value' x_0 can be chosen freely, each choice defining a particular equation (A 27); let us choose it in such a way that

$$2h(x_0) \geqslant h_\infty. \qquad (\text{A } 28)$$

With this choice, we shall easily prove that the operator acting on B in (A 27) is bounded. Since this operator is obviously triangular in time, it follows that (A 27) can be solved by a convergent series of successive time approximations.

A 4. *Boundedness of the operator in* (A 27)

Since R is a bounded operator on L_2, it is sufficient to prove the same for the operator $\mathbf{\Theta}$ whose kernel is

$$\mathbf{\Theta}(T,x_0;x,x_1) = \lim_{b\to 0}\int_{-\infty}^{+\infty} ds\, e^{-\pi s^2 b^2}\,\frac{\sin[\omega_0 T]}{\omega_0}\,k(x,x_0,s)\,\tilde{\eta}_0^0(s,x_1) \qquad (\text{A } 29a)$$

$$= 2\pi g\lim_{b\to 0}\int_{-\infty}^{+\infty} ds\, e^{-\pi s^2 b^2 - 2i\pi s X_s}\,\frac{\sinh[2\pi s\Delta] + i\Delta'\cosh[2\pi s\Delta]\sin[\omega_0 T]}{\cosh[2\pi s h_1]\cosh[2\pi s h_0]\,\omega_0}, \qquad (\text{A } 29b)$$

where $X = x - x_1$, $\Delta = h(x) - h(x_0)$, $\Delta' = h'(x)$, $h_1 = h(x_1)$, T is real, $\omega_0 = \omega(s, x_0)$. Thanks to (A 28), and since $h(x) < h_\infty$, the integral in (A 29) converges uniformly for any b, including $b = 0$.

For fixed values of Δ, Δ', h_0, h_1, the integral on the right-hand side of (A 29b) defines the Fourier transform of a function of s that is analytic in a disk centred at the origin. It therefore yields a function of X that goes to zero more rapidly than any power of X^{-1} as X goes to ∞. Since Δ, Δ', h_0, h_1 take their values on finite intervals, on which the integral convergence is uniform, it follows that $\mathbf{\Theta}$ can be bounded by $C(T)f(X)$, where $C(T)$ is a suitable continuous function of time, and $f(X)$ is a bounded function that goes to zero more rapidly than any power of X^{-1} as X goes to ∞. On the other hand, it is easily seen that $\mathbf{\Theta}$ vanishes when Δ and Δ' do, and thus in particular for $|x|$ and $|x_1|$ larger than x_∞. It follows from these results that $\|\mathbf{\Theta}\|_{L_2}$ is finite and vanishes with $\mu(h)$. This last result is sufficient to guarantee the convergence of the sequence

$$B_0(x,x_0,t) = A(x,t), \qquad (\text{A } 30a)$$

$$B_{n+1}(x,x_0,t) = A(x,t) - \int_0^t d\tau\, \Theta(t-\tau, x_0; x, x_1) \times \left[B_n(x_1,x_0,\tau) - \int_{-\infty}^{+\infty} R(x_1,x_2)\, B_n(x_2,x_0,\tau) \right]. \quad \text{(A 30}b\text{)}$$

Thus we construct $B(x,x_0,t)$, which is put for B in the right-hand side of (A 24), yielding $\tilde{\gamma}(s,x_0,t)$ in the left-hand side. From $\tilde{\gamma}(s,x_0,t)$ one obtains $\tilde{\eta}(s,t)$ by solving the Volterra equation (A 3), i.e.

$$\tilde{\eta}(s,t) = \tilde{\gamma}(s,x_0,t) - \omega_0^2 \int_0^t du\, \frac{\sin[\omega_0(t-u)]}{\omega_0}\, \tilde{\gamma}(s,x_0,u). \quad \text{(A 31)}$$

At this point, we have, in principle, completely solved the linear problem. We now calculate an approximate solution.

A 5. *The approximation used in the paper*

We keep at most the two first orders in t and $\|\epsilon\|$. From (A 31), (A 24), (A 30) we get

$$\tilde{\eta}(s,t) = \int_{-\infty}^{+\infty} dx\, \tilde{\eta}_0^0(s,x) \int_{-\infty}^{+\infty} dx_1 [\delta(x-x_1) - R(x,x_1)] \Big\{ A(x_1,t) - \omega^2(s,x_0) \int_0^t du (t-u)\, A(x_1,u) - \int_0^t du(t-u) \int_{-\infty}^{+\infty} ds'\, k(x_1,x_0,s') \times \int_{-\infty}^{+\infty} dx'\, \tilde{\eta}_0^0(s',x') \int_{-\infty}^{+\infty} [\delta(x',x_2) - R(x',x_2)]\, A(x_2,u) \Big\}. \quad \text{(A 32}a\text{)}$$

Now any function $G(s')$ can be put into the form $\int_{-\infty}^{+\infty} dx' \tilde{\eta}_0^0(s',x')\, \tilde{g}(x')$, since $\tilde{g}(x)$ is easily derived by solving

$$\tilde{g}(x) + \int_{-\infty}^{+\infty} dx'\, \epsilon(x,x')\, \tilde{g}(x') = \int_{-\infty}^{+\infty} ds'\, K(x,s')\, G(s').$$

Thus it is possible to calculate

$$\int_{-\infty}^{+\infty} dx\, \tilde{\eta}_0^0(s,x) \int_{-\infty}^{+\infty} dx_1\, [\delta(x-x_1) - R(x,x_1)]\, K(x_1,s')$$

by applying it to $\tilde{\eta}_0^0(s',x')$ so that the part of $k(x_1,x_0,s')$ that contains $\omega^2(s',x_0)$ cancels the other term containing ω_0^2 in (A 32). We knew this result for the exact solution with the condition (A 29). Here we see that it holds for any x_0, provided that the usual regularizations are applied. In particular, if $A(x,t) = A(x^*,t)\,\delta(x-x^*)$, it is very convenient to choose $x_0 = x^*$. Keeping only the two first non-vanishing orders in t and ϵ, we obtain

$$\tilde{\eta}(s,t) \approx \int_{-\infty}^{+\infty} dx\, \tilde{\eta}_0^0(s,x)\, [\delta(x-x_0) - \epsilon(x,x_0)] \left[A(x_0,t) - \omega^2(s,x_0) \int_0^t (t-u)\, A(x_0,u)\, du \right] - \int_0^t du(t-u)\, A(x_0,u) \int_{-\infty}^{+\infty} dx\, \tilde{\eta}_0^0(s,x) \int_{-\infty}^{+\infty} ds'\, k(x,x_0,s')\, \tilde{\eta}_0^0(s',x_0). \quad \text{(A 32}b\text{)}$$

The last term in the right-hand side of (A 32b) contains the integral

$$\theta_0(x,x_0) = ig \lim_{b\to 0} \int_{-\infty}^{+\infty} ds\, e^{-\pi s^2 b^2}\, \frac{\partial}{\partial x} \frac{\sinh[2\pi s(h(x) - h(x_0))]}{\cosh^2[2\pi s h(x_0)]}\, e^{-2i\pi s(x-x_0)} \quad \text{(A 33)}$$

The calculation of θ_0 can be regularized with the help of the identity

$$\frac{\cosh H_1}{\cosh^{k+2} H_0} = \sum_{p=1}^{n-k} a_p^{(k)} \cosh [H_1 - (2p+k) H_0]$$

$$+(-1)^{n+1}\left\{b_n^{k+1}\frac{\cosh [H_1-(2n-k+1)H_0]}{\cosh H_0}+b_n^k\frac{\cosh [H_1-(2n-k+2)H_0]}{\cosh^2 H_0}\right.$$

$$\left.+\dots+b_n^1\frac{\cosh [H_1-(2n+1)H_0]}{\cosh^{k+1} H_0}+\frac{\cosh [H_1-(2n+2)H_0]}{\cosh^{k+2} H_0}\right\}, \quad \text{(A 34)}$$

where

$$a_p^{(k)} = 2^{k+2}(-1)^{p+1}\frac{\Gamma(p+1+k)}{\Gamma(p)\,\Gamma(k+2)},$$

$$b_n^{k+1} = 2^{k+1}(-1)^{k+1}\frac{\Gamma(2+n)}{\Gamma(1+n-k)\,\Gamma(k+2)}.$$

This formula can be proved by induction on n and k, which yields the recurrences $a_p^{(k)}+a_{p-1}^{(k)} = 2a_p^{(k-1)}$; $a_{n-k}^{(k)} = 2(-1)^n b_{n-1}^{k+1}$; $b_n^p + 2b_{n-1}^{p-1} = b_{n-1}^p$; with $b_n^0 = 1$ and $a_0^{(k)} = 0$. A quite-similar formula is obtained for the hyperbolic sines. They reduce to (A 7) and (A 8) for $k=-1$. The identity (A 34) is to be used (here for $k=0$) as we did with (A 6) when we derived $\epsilon(x, x_0)$. The contribution of the term 'without denominator' vanishes at the limit $b=0$. The remainder yields

$$\theta_0 = \frac{1}{\pi h_0}\frac{\partial}{\partial x}\cdot\frac{\bar{X}\cosh\bar{X}\sin\bar{\Delta}-\bar{\Delta}\sinh\bar{X}\cos\bar{\Delta}}{\sin^2\bar{\Delta}+\sinh^2\bar{X}}, \quad \text{(A 35)}$$

where $\bar{X} = \frac{1}{2}\pi(x-x_0)/h_0$, $\bar{\Delta} = \frac{1}{2}\pi[h(x)-h_0]/h_0$. Hence θ_0 goes to zero with $\mu(h)$, and we can write down the response (A 32) to the source $A(x_0,t)\,\delta(x-x_0)$ as

$$\tilde{\eta}(s,t) = \left[A(x_0,t)-\omega^2(s,x_0)\int_0^t du(t-u)\,A(x_o,u)\right]\left[\tilde{\eta}_0^0(s,x_0)\right.$$

$$\left.-\int_{-\infty}^{+\infty} dx\,\tilde{\eta}_0^0(s,x)\,\epsilon(x,x_0)\right]-\int_0^t du(t-u)\,A(x_0,u)\int_{-\infty}^{+\infty} dx$$

$$\times\,\tilde{\eta}_0^0(s,x)\,\theta_0(x,x_0)+[O(t^4)+O(\mu^2(h))]\,O(A(x_0,t)). \quad \text{(A 36)}$$

Notice that the term containing θ_0 is of relative order $t^2\mu(h)$. If we keep only the two first time orders, we have a response to $A(x_0,t)\;\delta(x-x_0)$ on a sloping bottom equal to that to $A(x_0,t)\;[\delta(x-x_0)-\epsilon(x,x_0)]$ on a horizontal one. It is easy to show that the two sources have the same area. For deriving (4.1) we have used this equivalence and replaced the two first orders of the time series by a cosine, obtaining in such a way a result that is similar with the 'intuitive' result obtained from (3.5).

A 6. *Three-dimensional case*

From (2.3) or (2.12) we see that we have to solve

$$\left[\left(\frac{\partial}{\partial y}+h'(x)\frac{\partial}{\partial x}\right)\int_{-\infty}^{+\infty}\exp\,[-2i\pi s x]\left(k^{-1}\sinh\,[ky]\,\tilde{\eta}'(k,s,x_0,t)\right.\right.$$

$$\left.\left.-g\cosh\,[ky]\int_0^t (t-\tau)\,\tilde{\eta}'(k,s,x_0,\tau)\,d\tau\right)\right]_{y=-h(x)} = \delta(x-x_0)\,A(x_0,t) \quad \text{(A 37)}$$

for the 'direct' term (separating the direct and the reflected terms is as in the two-dimensional case). Let us introduce the function

$$\tilde{\gamma}(k,s,x_0,t) = \tilde{\eta}'(k,s,x_0,t) + \omega_0^2(k)\int_0^t (t-\tau)\,\tilde{\eta}'(k,s,x_0,\tau)\,d\tau,$$

where $\omega_0(k)$ is given by (3.3). We obtain from (A 37) the equation

$$\lim_{b\to 0}\left\{\left(\frac{\partial}{\partial y}+h'(x)\frac{\partial}{\partial x}\right)\int_{-\infty}^{+\infty} ds\exp\left[-2i\pi sx-\frac{b^2k^2}{4\pi}\right]\left[\frac{\sinh[ky]}{k}\right.\right.$$
$$\left.\left.\times\tilde{\gamma}(k,s,x_0,t)-g\,\frac{\cosh[k(y+h_0)]}{\cosh[kh_0]}\int_0^t du\,\frac{\sin[\omega_0(t-u)]}{\omega_0}\,\tilde{\gamma}(k,s,x_0,u)\right]\right\}_{y=-h(x)}$$
$$= \delta(x-x_0)\,A(x_0,t). \quad \text{(A 38)}$$

The equation (A 38) is formally similar to (A 4). It can be managed by the same method, provided that we get for $\tilde{\gamma} = \exp[2i\pi sx_0]\operatorname{sech}[kh_0]\,A(x_0,t)$ again the response $A(x_0,t)\,[\delta(x-x_0)+\epsilon(x,x_0)]$, where $\|\epsilon\|$ is $O(\mu(h))$. Hence we have to calculate

$$H_b(x,x_0) = \left\{\left(\frac{\partial}{\partial y}+h'(x)\frac{\partial}{\partial x}\right)\int_{-\infty}^{+\infty} ds\exp[-2i\pi s(x-x_0)]f_b(k,y,x_0)\right\}_{y=-h(x)}, \quad \text{(A 39)}$$

where
$$f_b'(k,y,x_0) \equiv \exp\left[-\frac{b^2k^2}{4\pi}\right](k\cosh[kh_0])^{-1}\sinh[ky]$$
$$= \pi^{-1}\int_0^\infty du\,k^{-1}\sin[ku]\,F_b'(u,y,x_0) \quad \text{(A 40)}$$

$\left(\text{so that } F_b'(u,y,x_0) = 2\int_0^\infty dk\,k\sin[ku]f_b'(k,y,x_0)\right)$. Inserting (A 40) into (A 39) we get

$$2\pi H_b(x,x_0) = \left\{\left(\frac{\partial}{\partial y}+h'(x)\frac{\partial}{\partial x}\right)\int_{|X|}^{\infty} du\,F_b'(u,y,x_0)\,J_0[2\pi r(u^2-X^2)^{\frac{1}{2}}]\right\}_{y=-h(x)}, \quad \text{(A 41)}$$

where $X = x-x_0$. Let us now introduce the function

$$F_b(u,y,x_0) = 2\int_0^\infty \cos ku\,f_b(k,y,x_0)\,dk, \quad \text{(A 42a)}$$

where
$$f_b(k,y,x_0) = \frac{\partial}{\partial y}f_b'(k,y,x_0). \quad \text{(A 42b)}$$

Clearly $\partial F_b'/\partial y = -\partial F_b/\partial u$. Using this equality in the first term of (A 41), and integrating by parts, we obtain

$$2\pi H_b(x,x_0) = F_b(|X|,-h(x),x_0) - h'(x)\frac{\partial|X|}{\partial x}F_b'(|X|,-h(x),x_0)$$
$$-\int_X^\infty du\left[F_b(u,-h(x),x_0)-h'(x)\frac{X}{u}F_b'(u,-h(x),x_0)\right]\frac{2\pi ruJ_1[2\pi r(u^2-X^2)^{\frac{1}{2}}]}{(u^2-X^2)^{\frac{1}{2}}}. \quad \text{(A 43)}$$

The first two terms in (A 43), which together constitute the 'free' part of $H_b(x,x_0)$, say $H_b^{\mathrm{f}}(x,x_0)$, are equal to

$$2\pi H_b^{\mathrm{f}}(x,x_0) = \int_{-\infty}^{+\infty} dk\exp\left[-ikX-\frac{b^2k^2}{4\pi}\right]\operatorname{sech}^2 kh_0$$
$$\times[\cosh[kh(x)]+ih'(x)\sinh[kh(x)]], \quad \text{(A 44)}$$

so that $H_0^\xi(x, x_0) = \delta(x-x_0)+\epsilon(x, x_0)$, which is the desired result. The integrated terms in (A 43) can be regularized and calculated by the methods introduced in the two-dimensional case. They yield higher-order terms. In the present paper, we use mainly

$$\tilde{\gamma}_0(k, s, x_0, t) = \exp[2i\pi s x_0]\operatorname{sech}[kh_0]A(x_0, t), \tag{A 45}$$

which yields the 'source' $\delta(x-x_0)-O(\bar{h}')$, and is therefore the zeroth-order solution. We also use a 'first-order' solution, which is meant as follows: gt^2/h and h' are considered equivalent infinitesimal quantities of first order. Thus the three-dimensional equivalents of only the first two terms in (A 36) remain, and, going to $\tilde{\eta}'(k, s, x_0, t)$, we obtain

$$\tilde{\eta}'(k, s, x_0, t) = \tilde{\gamma}_0(k, s, x_0, t) - \int_{-\infty}^{+\infty} dx_1\, \tilde{\gamma}_0(k, s, x_1, t)\,\varepsilon(x_1, x_0) - \omega_0^2(k)\int_0^t (t-u) \times \tilde{\gamma}_0(k, s, x_0, u)\,du; \tag{A 46}$$

$$\varepsilon(x, x_0) = \epsilon_0(x, x_0) + \frac{\pi r}{h_0}\int_{|X|}^{\infty} du\, \frac{J_1[2\pi r(u^2-X^2)^{\frac{1}{2}}]}{(u^2-X^2)^{\frac{1}{2}}} \times \frac{u\cosh\dfrac{\pi u}{2h_0}\sin\dfrac{\pi\Delta_0}{2h_0} - Xh'(x)\sinh\dfrac{\pi u}{2h_0}\cos\dfrac{\pi\Delta_0}{2h_0}}{\sin^2\dfrac{\pi\Delta_0}{2h_0}+\sinh^2\dfrac{\pi u}{2h_0}}; \tag{A 47}$$

$\epsilon_0(x, x_0)$ is given by (A 13). Keeping only in ε and in the other terms of (A 43) the terms that are $O(h')$ we obtain

$$\varepsilon(x, x_0) \approx -\frac{h_0' X}{2h_0}\int_{|X|}^{\infty} J_0[2\pi r(u^2-X^2)^{\frac{1}{2}}]\frac{\partial}{\partial u}[u^{-1}f(u)]fu, \tag{A 48}$$

where

$$f(u) = \operatorname{cosech}^2\frac{\pi u}{2h_0}\left\{\sinh\frac{\pi u}{2h_0} - \frac{\pi u}{2h_0}\cosh\frac{\pi u}{2h_0}\right\} \simeq -\frac{\pi u}{4h_0}\exp\left[-\frac{4}{15}\left(\frac{\pi u}{2h_0}\right)^2\right] \quad \text{for} \quad u/h_0 \lesssim 1.$$

A 7. *Well-posedness proofs*

(i) In this sketch of a proof, we consider the finite domain obtained by making $h(x)$ go to zero for some large value of x, $x = \pm\alpha$, with infinite derivatives up to and including the fourth order. We can continue this mathematical domain into $y > 0$ by symmetry with respect to the x-axis. Let the whole boundary $\mathscr{B}$ be parametrized by the curve abscissa s. We assume that $(1+h'^2(x))^{-\frac{1}{2}}A(x) = \alpha(s)$ is twice differentiable and is continued by reflection, together with Laplace's equation. Thus we get a single-layer potential $\Psi(x, y)$, which is an odd function of y:

$$\Psi(x, y) = -\int_{\mathscr{B}} \tau(s')\log[r(x, y; s')]\,ds', \tag{A 49}$$

where $r(x, y; s')$ is the distance between any point (x, y) of the domain and the point of abscissa s' on $\mathscr{B}$. It follows from (2.1)–(2.4′) that the response at $t = 0^+$ is $[\partial\Psi(x, y)/\partial y]_{y=0}$, which is related to $\tau(s)$ through (A 45). The jump discontinuity theorem yields the equation replacing (2.16), and whose solution is $\tau(s)$:

$$-\alpha(s) = \pi\tau(s) - \int_{\mathscr{B}} \tau(s')\frac{\partial}{\partial\nu_s}\log[r(s, s')]\,ds', \tag{A 50}$$

where $\partial/\partial\nu_s$ is the normal derivative on $\mathscr{B}$ at the point of abscissa s. It is well-known that the kernel of this Fredholm equation is continuous (Courant & Hilbert 1962).

Hence the Fredholm alternative holds. One shows that the homogeneous equation has only the zero solution either by using the Dirichlet integral or by discarding directly the possibility of π as an eigenvalue (as in Courant & Hilbert). Hence the resolvent exists as a bounded operator, and τ depends continuously on α. Besides, $\tau(s)$ is twice differentiable if $\alpha(s)$ is, so that $(\partial \Psi/\partial y)_{y=0}$ is well-defined. Notice that this derivation can obviously be used to define a numerical method for obtaining 'exactly' the surface deformation at $t = 0^+$.

(ii) Thus we have proved the well-posedness for a finite basin and the sudden approximation. But what about the general case? In this paper we have seen that the regularized formulation can be managed by successive approximations in a way to give a solution that depends continuously on the data. However, the regularization trick is justified in one way only: we have proved that *if* there exists a solution $\eta(x)$, and $\tilde{\eta}(s)$ is multiplied by $\exp[-\pi b^2 s^2]$, with $b \to 0$, this $\tilde{\eta}(s)$ can be determined as a solution of (2.13) and *if* it is of the form (2.16), then $\sigma(x)$ is a solution of (2.15). For an infinite basin we have neither proved the existence and uniqueness of a solution of (2.15) nor the uniqueness of a solution of the linear problem, two missing steps. On the other hand, our assumptions are very strong, and we suspect that if the bottom is very irregular, with overhanging cliffs and holes in which the water can get uncontrolled accelerations, the motion cannot be irrotational. But our algorithms can give a rough approximation, as long as Assumptions A and B have some physical support.

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